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6

A small buck from the massive bull transfers a large amount of momentum to the cowboy, resulting in an involuntary dismount.

MOMENTUM AND COLLISIONS

What happens when two automobiles collide? How does the impact affect the motion of each vehicle, and what basic physical principles determine the likelihood of serious injury? How do rockets work, and what mechanisms can be used to overcome the limitations imposed by exhaust speed? Why do we have to brace ourselves when firing small projectiles at high velocity? Finally, how can we use physics to improve our golf game?

To begin answering such questions, we introduce *momentum*. Intuitively, anyone or anything that has a lot of momentum is going to be hard to stop. In politics, the term is metaphorical. Physically, the more momentum an object has, the more force has to be applied to stop it in a given time. This concept leads to one of the most powerful principles in physics: *conservation of momentum*. Using this law, complex collision problems can be solved without knowing much about the forces involved during contact. We'll also be able to derive information about the average force delivered in an impact. With conservation of momentum, we'll have a better understanding of what choices to make when designing an automobile or a moon rocket, or when addressing a golf ball on a tee.

6.1 MOMENTUM AND IMPULSE

In physics, momentum has a precise definition. A slowly moving brontosaurus has a lot of momentum, but so does a little hot lead shot from the muzzle of a gun. We therefore expect that momentum will depend on an object's mass and velocity.

← Linear momentum

The linear momentum $\vec{\mathbf{p}}$ of an object of mass m moving with velocity $\vec{\mathbf{v}}$ is the product of its mass and velocity:

$$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}} \qquad [6.1]$$

SI unit: kilogram-meter per second (kg·m/s)

Doubling either the mass or the velocity of an object doubles its momentum; doubling both quantities quadruples its momentum. Momentum is a vector quantity with the same direction as the object's velocity. Its components are given in two dimensions by

$$p_x = mv_x \qquad p_y = mv_y$$

where p_x is the momentum of the object in the x-direction and p_y its momentum in the y-direction.

The magnitude of the momentum p of an object of mass m can be related to its kinetic energy KE:

$$KE = \frac{p^2}{2m} \qquad [6.2]$$

This relationship is easy to prove using the definitions of kinetic energy and momentum (see Problem 6.6) and is valid for objects traveling at speeds much less than the speed of light. Equation 6.2 is useful in grasping the interplay between the two concepts, as illustrated in Quick Quiz 6.1.

QUICK QUIZ 6.1 Two masses m_1 and m_2, with $m_1 < m_2$, have equal kinetic energy. How do the magnitudes of their momenta compare? (a) Not enough information is given. (b) $p_1 < p_2$ (c) $p_1 = p_2$ (d) $p_1 > p_2$

Changing the momentum of an object requires the application of a force. This is, in fact, how Newton originally stated his second law of motion. Starting from the more common version of the second law, we have

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = m\frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\Delta(m\vec{\mathbf{v}})}{\Delta t}$$

where the mass m and the forces are assumed constant. The quantity in parentheses is just the momentum, so we have the following result:

The change in an object's momentum $\Delta\vec{\mathbf{p}}$ divided by the elapsed time Δt equals the constant net force $\vec{\mathbf{F}}_{\text{net}}$ acting on the object:

Newton's second law and momentum →

$$\frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{\text{change in momentum}}{\text{time interval}} = \vec{\mathbf{F}}_{\text{net}} \qquad [6.3]$$

This equation is also valid when the forces are not constant, provided the limit is taken as Δt becomes infinitesimally small. Equation 6.3 says that if the net force on an object is zero, the object's momentum doesn't change. In other words, the linear momentum of an object is conserved when $\vec{\mathbf{F}}_{\text{net}} = 0$. Equation 6.3 also tells us that changing an object's momentum requires the continuous application of a force over a period of time Δt, leading to the definition of *impulse*:

If a constant force $\vec{\mathbf{F}}$ acts on an object, the **impulse** $\vec{\mathbf{I}}$ delivered to the object over a time interval Δt is given by

$$\vec{\mathbf{I}} \equiv \vec{\mathbf{F}}\,\Delta t \qquad [6.4]$$

SI unit: kilogram meter per second (kg·m/s)

Impulse is a vector quantity with the same direction as the constant force acting on the object. When a single constant force $\vec{\mathbf{F}}$ acts on an object, Equation 6.3 can be written as

Impulse–momentum theorem →

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}\,\Delta t = \Delta\vec{\mathbf{p}} = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i \qquad [6.5]$$

This is a special case of the **impulse–momentum theorem**. Equation 6.5 shows that **the impulse of the force acting on an object equals the change in momentum of that object**. This equality is true even if the force is not constant, as long as the time interval Δt is taken to be arbitrarily small. (The proof of the general case requires concepts from calculus.)

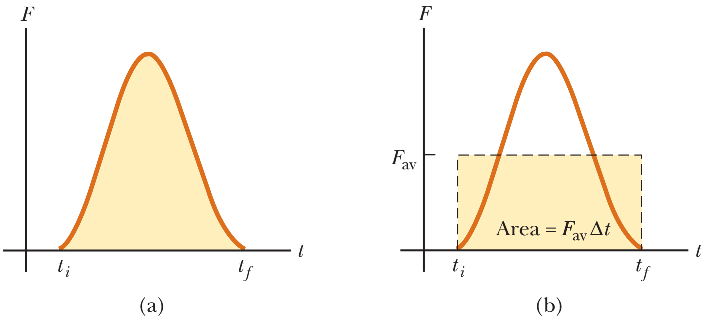


FIGURE 6.1 (a) A force acting on an object may vary in time. The impulse is the area under the force vs. time curve. (b) The average force (horizontal dashed line) gives the same impulse to the object in the time interval Δt as the real time-varying force described in (a).

In real-life situations, the force on an object is only rarely constant. For example, when a bat hits a baseball, the force increases sharply, reaches some maximum value, and then decreases just as rapidly. Figure 6.1(a) shows a typical graph of force versus time for such incidents. The force starts out small as the bat comes in contact with the ball, rises to a maximum value when they are firmly in contact, and then drops off as the ball leaves the bat. In order to analyze this rather complex interaction, it's useful to define an **average force** $\vec{\mathbf{F}}_{av}$, shown as the dashed line in Figure 6.1b. This average force is the constant force delivering the same impulse to the object in the time interval Δt as the actual time-varying force. We can then write the impulse–momentum theorem as

$$\vec{\mathbf{F}}_{av}\,\Delta t = \Delta\vec{\mathbf{p}} \qquad \textbf{[6.6]}$$

The magnitude of the impulse delivered by a force during the time interval Δt is equal to the area under the force vs. time graph as in Figure 6.1a or, equivalently, to $F_{av}\,\Delta t$ as shown in Figure 6.1b. The brief collision between a bullet and an apple is illustrated in Figure 6.2.

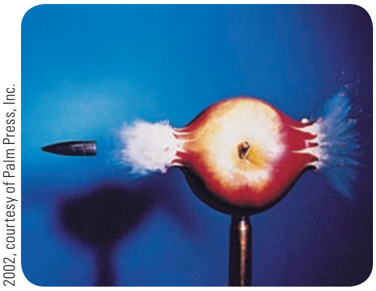


FIGURE 6.2 An apple being pierced by a 30-caliber bullet traveling at a supersonic speed of 900 m/s. This collision was photographed with a microflash stroboscope using an exposure time of 0.33 μs. Shortly after the photograph was taken, the apple disintegrated completely. Note that the points of entry and exit of the bullet are visually explosive.

APPLYING PHYSICS 6.1 BOXING AND BRAIN INJURY

In boxing matches of the 19th century, bare fists were used. In modern boxing, fighters wear padded gloves. How do gloves protect the brain of the boxer from injury? Also, why do boxers often "roll with the punch"?

Explanation The brain is immersed in a cushioning fluid inside the skull. If the head is struck suddenly by a bare fist, the skull accelerates rapidly. The brain matches this acceleration only because of the large impulsive force exerted by the skull on the brain. This large and sudden force (large F_{av} and small Δt) can cause severe brain injury. Padded gloves extend the time Δt over which the force is applied to the head. For a given impulse $F_{av}\Delta t$, a glove results in a longer time interval than a bare fist, decreasing the average force. Because the average force is decreased, the acceleration of the skull is decreased, reducing (but not eliminating) the chance of brain injury. The same argument can be made for "rolling with the punch": If the head is held steady while being struck, the time interval over which the force is applied is relatively short and the average force is large. If the head is allowed to move in the same direction as the punch, the time interval is lengthened and the average force reduced.

EXAMPLE 6.1 Teeing Off

Goal Use the impulse–momentum theorem to estimate the average force exerted during an impact.

Problem A golf ball with mass 5.0×10^{-2} kg is struck with a club as in Figure 6.3. The force on the ball varies from zero when contact is made up to some maximum value (when the ball is maximally deformed) and then back to zero when the ball leaves the club, as in the graph of force vs. time in Figure 6.1. Assume that the ball leaves the club face with a velocity of +44 m/s. **(a)** Find the magnitude of the impulse due to the collision. **(b)** Estimate the duration of the collision and the average force acting on the ball.

Strategy In part (a), use the fact that the impulse is equal to the change in momentum. The mass and the initial and final speeds are known, so this change can be computed. In part (b), the average force is just the change in momentum computed in part (a) divided by an estimate of the duration of the collision. Guess at the distance the ball travels on the face of the club (about 2 cm, roughly the same as the radius of the ball). Divide this distance by the average velocity (half the final velocity) to get an estimate of the time of contact.

FIGURE 6.3 (Example 6.1) A golf ball being struck by a club.

Solution

(a) Find the impulse delivered to the ball.

The problem is essentially one dimensional. Note that $v_i = 0$, and calculate the change in momentum, which equals the impulse:

$$I = \Delta p = p_f - p_i = (5.0 \times 10^{-2}\ \text{kg})(44\ \text{m/s}) - 0$$
$$= \boxed{+2.2\ \text{kg} \cdot \text{m/s}}$$

(b) Estimate the duration of the collision and the average force acting on the ball.

Estimate the time interval of the collision, Δt, using the approximate displacement (radius of the ball) and its average speed (half the maximum speed):

$$\Delta t = \frac{\Delta x}{v_{\text{av}}} = \frac{2.0 \times 10^{-2}\ \text{m}}{22\ \text{m/s}} = \boxed{9.1 \times 10^{-4}\ \text{s}}$$

Estimate the average force from Equation 6.6:

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{2.2\ \text{kg}\cdot\text{m/s}}{9.1 \times 10^{-4}\ \text{s}} = \boxed{+2.4 \times 10^{3}\ \text{N}}$$

Remarks This estimate shows just how large such contact forces can be. A good golfer achieves maximum momentum transfer by shifting weight from the back foot to the front foot, transmitting the body's momentum through the shaft and head of the club. This timing, involving a short movement of the hips, is more effective than a shot powered exclusively by the arms and shoulders. Following through with the swing ensures that the motion isn't slowed at the critical instant of impact.

QUESTION 6.1

What average club speed would double the average force?

EXERCISE 6.1

A 0.150-kg baseball, thrown with a speed of 40.0 m/s, is hit straight back at the pitcher with a speed of 50.0 m/s. (a) What is the impulse delivered by the bat to the baseball? (b) Find the magnitude of the average force exerted by the bat on the ball if the two are in contact for 2.00×10^{-3} s.

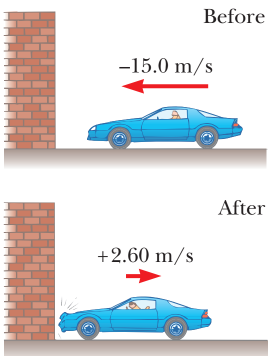
Answer (a) 13.5 kg · m/s (b) 6.75 kN

EXAMPLE 6.2 How Good Are the Bumpers?

Goal Find an impulse and estimate a force in a collision of a moving object with a stationary object.

Problem In a crash test, a car of mass 1.50×10^3 kg collides with a wall and rebounds as in Figure 6.4a. The initial and final velocities of the car are $v_i = -15.0$ m/s and $v_f = 2.60$ m/s, respectively. If the collision lasts for 0.150 s, find (a) the impulse delivered to the car due to the collision and (b) the size and direction of the average force exerted on the car.

Strategy This problem is similar to the previous example, except that the initial and final momenta



(a)

(b)

FIGURE 6.4 (Example 6.2) (a) This car's momentum changes as a result of its collision with the wall. (b) In a crash test (an inelastic collision), much of the car's initial kinetic energy is transformed into the energy it took to damage the vehicle.

are both nonzero. Find the momenta and substitute into the impulse–momentum theorem, Equation 6.6, solving for F_{av}.

Solution

(a) Find the impulse delivered to the car.

Calculate the initial and final momenta of the car:

$$p_i = mv_i = (1.50 \times 10^3 \text{ kg})(-15.0 \text{ m/s})$$
$$= -2.25 \times 10^4 \text{ kg} \cdot \text{m/s}$$
$$p_f = mv_f = (1.50 \times 10^3 \text{ kg})(+2.60 \text{ m/s})$$
$$= +0.390 \times 10^4 \text{ kg} \cdot \text{m/s}$$

The impulse is just the difference between the final and initial momenta:

$$\text{I} = p_f - p_i$$
$$= +0.390 \times 10^4 \text{ kg} \cdot \text{m/s} - (-2.25 \times 10^4 \text{ kg} \cdot \text{m/s})$$
$$I = \boxed{2.64 \times 10^4 \text{ kg} \cdot \text{m/s}}$$

(b) Find the average force exerted on the car.

Apply Equation 6.6, the impulse–momentum theorem:

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{2.64 \times 10^4 \text{ kg} \cdot \text{m/s}}{0.150 \text{ s}} = \boxed{+1.76 \times 10^5 \text{ N}}$$

Remarks When the car doesn't rebound off the wall, the average force exerted on the car is smaller than the value just calculated. With a final momentum of zero, the car undergoes a smaller change in momentum.

QUESTION 6.2

When a person is involved in a car accident, why is the likelihood of injury greater in a head-on collision as opposed to being hit from behind? Answer using the concepts of relative velocity, momentum, and average force.

EXERCISE 6.2

Suppose the car doesn't rebound off the wall, but the time interval of the collision remains at 0.150 s. In this case, the final velocity of the car is zero. Find the average force exerted on the car.

Answer $+1.50 \times 10^5$ N

Injury in Automobile Collisions

APPLICATION

Injury to Passengers in Car Collisions

The main injuries that occur to a person hitting the interior of a car in a crash are brain damage, bone fracture, and trauma to the skin, blood vessels, and internal organs. Here, we compare the rather imprecisely known thresholds for human injury with typical forces and accelerations experienced in a car crash.

A force of about 90 kN (20 000 lb) compressing the tibia can cause fracture. Although the breaking force varies with the bone considered, we may take this value as the threshold force for fracture. It's well known that rapid acceleration of the head, even without skull fracture, can be fatal. Estimates show that head accelerations of $150g$ experienced for about 4 ms or $50g$ for 60 ms are fatal 50% of the time. Such injuries from rapid acceleration often result in nerve damage to the spinal cord where the nerves enter the base of the brain. The threshold for damage to skin, blood vessels, and internal organs may be estimated from whole-body impact data, where the force is uniformly distributed over the entire front surface area of 0.7 m^2 to 0.9 m^2. These data show that if the collision lasts for less than about 70 ms, a person will survive if the whole-body impact pressure (force per unit area) is less than 1.9×10^5 N/m^2 (28 lb/in.^{2}). Death results in 50% of cases in which the whole-body impact pressure reaches 3.4×10^5 N/m^2 (50 lb/in.^{2}).

Armed with the data above, we can estimate the forces and accelerations in a typical car crash and see how seat belts, air bags, and padded interiors can reduce the chance of death or serious injury in a collision. Consider a typical collision involving a 75-kg passenger not wearing a seat belt, traveling at 27 m/s (60 mi/h) who comes to rest in about 0.010 s after striking an unpadded dashboard. Using $F_{av}\,\Delta t = mv_f - mv_i$, we find that

$$F_{av} = \frac{mv_f - mv_i}{\Delta t} = \frac{0 - (75\text{ kg})(27\text{ m/s})}{0.010\text{ s}} = -2.0 \times 10^5\text{ N}$$

and

$$a = \left|\frac{\Delta v}{\Delta t}\right| = \frac{27\text{ m/s}}{0.010\text{ s}} = 2\,700\text{ m/s}^2 = \frac{2\,700\text{ m/s}^2}{9.8\text{ m/s}^2}\,g = 280g$$

If we assume the passenger crashes into the dashboard and windshield so that the head and chest, with a combined surface area of 0.5 m^2, experience the force, we find a whole-body pressure of

$$\frac{F_{av}}{A} = \frac{2.0 \times 10^5\text{ N}}{0.5\text{ m}^2} \cong 4 \times 10^5\text{ N/m}^2$$

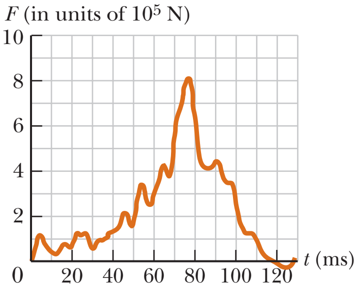


FIGURE 6.5 Force on a car versus time for a typical collision.

We see that the force, the acceleration, and the whole-body pressure all *exceed* the threshold for fatality or broken bones and that an unprotected collision at 60 mi/h is almost certainly fatal.

What can be done to reduce or eliminate the chance of dying in a car crash? The most important factor is the collision time, or the time it takes the person to come to rest. If this time can be increased by 10 to 100 times the value of 0.01 s for a hard collision, the chances of survival in a car crash are much higher because the increase in Δt makes the contact force 10 to 100 times smaller. Seat belts restrain people so that they come to rest in about the same amount of time it takes to stop the car, typically about 0.15 s. This increases the effective collision time by an order of magnitude. Figure 6.5 shows the measured force on a car versus time for a car crash.

Air bags also increase the collision time, absorb energy from the body as they rapidly deflate, and spread the contact force over an area of the body of about 0.5 m^2, preventing penetration wounds and fractures. Air bags must deploy very rapidly (in less than 10 ms) in order to stop a human traveling at 27 m/s before he or she comes to rest against the steering column about 0.3 m away. To achieve this rapid deployment, accelerometers send a signal to discharge a bank of capacitors (devices that store electric charge), which then ignites an explosive, thereby filling the air bag with gas very quickly. The electrical charge for ignition is stored in capacitors to ensure that the air bag continues to operate in the event of damage to the battery or the car's electrical system in a severe collision.

The important reduction in potentially fatal forces, accelerations, and pressures to tolerable levels by the simultaneous use of seat belts and air bags is summarized as follows: If a 75-kg person traveling at 27 m/s is stopped by a seat belt in 0.15 s, the person experiences an average force of 9.8 kN, an average acceleration of 18g, and a whole-body pressure of 2.8×10^4 N/m^2 for a contact area of 0.5 m^2. These values are about one order of magnitude less than the values estimated earlier for an unprotected person and well below the thresholds for life-threatening injuries.

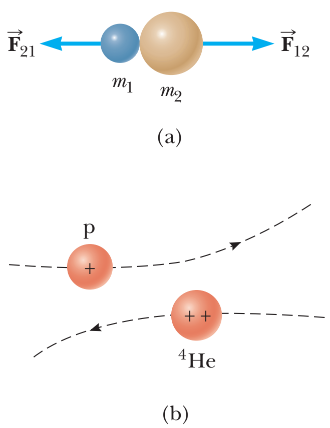


FIGURE 6.6 (a) A collision between two objects resulting from direct contact. (b) A collision between two charged objects (in this case, a proton and a helium nucleus).

6.2 CONSERVATION OF MOMENTUM

When a collision occurs in an isolated system, the total momentum of the system doesn't change with the passage of time. Instead, it remains constant both in magnitude and in direction. The momenta of the individual objects in the system may change, but the vector sum of *all* the momenta will not change. The total momentum is therefore said to be *conserved*. In this section, we will see how the laws of motion lead us to this important conservation law.

A collision may be the result of physical contact between two objects, as illustrated in Figure 6.6a. This is a common macroscopic event, as when a pair of billiard balls or a baseball and a bat strike each other. By contrast, because contact on a submicroscopic scale is hard to define accurately, the notion of *collision* must be generalized to that scale. Forces between two objects arise from the electrostatic interaction of the electrons in the surface atoms of the objects. As will be discussed in Chapter 15, electric charges are either positive or negative. Charges with the same sign repel each other, while charges with opposite sign attract each other. To understand the distinction between macroscopic and microscopic collisions, consider the collision between two positive charges, as shown in Figure 6.6b. Because the two particles in the figure are both positively charged, they repel each other. During such a microscopic collision, particles need not touch in the normal sense in order to interact and transfer momentum.

Active Figure 6.7 shows an isolated system of two particles before and after they collide. By "isolated," we mean that no external forces, such as the gravitational force or friction, act on the system. Before the collision, the velocities of the two particles are $\vec{\mathbf{v}}_{1i}$ and $\vec{\mathbf{v}}_{2i}$; after the collision, the velocities are $\vec{\mathbf{v}}_{1f}$ and $\vec{\mathbf{v}}_{2f}$. The impulse–momentum theorem applied to m_1 becomes

$$\vec{\mathbf{F}}_{21}\,\Delta t = m_1\vec{\mathbf{v}}_{1f} - m_1\vec{\mathbf{v}}_{1i}$$

Likewise, for m_2, we have

$$\vec{\mathbf{F}}_{12}\,\Delta t = m_2\vec{\mathbf{v}}_{2f} - m_2\vec{\mathbf{v}}_{2i}$$

where $\vec{\mathbf{F}}_{21}$ is the average force exerted by m_2 on m_1 during the collision and $\vec{\mathbf{F}}_{12}$ is the average force exerted by m_1 on m_2 during the collision, as in Figure 6.6a.

We use average values for $\vec{\mathbf{F}}_{21}$ and $\vec{\mathbf{F}}_{12}$ even though the actual forces may vary in time in a complicated way, as is the case in Figure 6.8. Newton's third law states that at all times these two forces are equal in magnitude and opposite in direction: $\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12}$. In addition, the two forces act over the same time interval. As a result, we have

$$\vec{\mathbf{F}}_{21}\,\Delta t = -\vec{\mathbf{F}}_{12}\,\Delta t$$

or

$$m_1\vec{\mathbf{v}}_{1f} - m_1\vec{\mathbf{v}}_{1i} = -(m_2\vec{\mathbf{v}}_{2f} - m_2\vec{\mathbf{v}}_{2i})$$

after substituting the expressions obtained for $\vec{\mathbf{F}}_{21}$ and $\vec{\mathbf{F}}_{12}$. This equation can be rearranged to give the following important result:

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} \qquad \textbf{[6.7]}$$

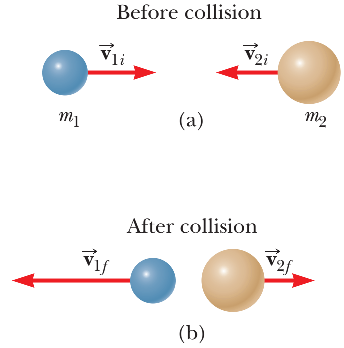
This result is a special case of the law of **conservation of momentum** and is true of isolated systems containing any number of interacting objects.

← Conservation of momentum

When no net external force acts on a system, the total momentum of the system remains constant in time.

Defining the isolated system is an important feature of applying this conservation law. A cheerleader jumping upwards from rest might appear to violate conservation of momentum, because initially her momentum is zero and suddenly she's leaving the ground with velocity $\vec{\mathbf{v}}$. The flaw in this reasoning lies in the fact that the cheerleader isn't an isolated system. In jumping, she exerts a downward force on the Earth, changing its momentum. This change in the Earth's momentum isn't noticeable, however, because of the Earth's gargantuan mass compared to the cheerleader's. When we define the system to be *the cheerleader and the Earth*, momentum is conserved.

Action and reaction, together with the accompanying exchange of momentum between two objects, is responsible for the phenomenon known as *recoil*. Everyone knows that throwing a baseball while standing straight up, without bracing your feet against the Earth, is a good way to fall over backwards. This reaction, an



ACTIVE FIGURE 6.7
Before and after a head-on collision between two objects. The momentum of each object changes as a result of the collision, but the total momentum of the system remains constant.

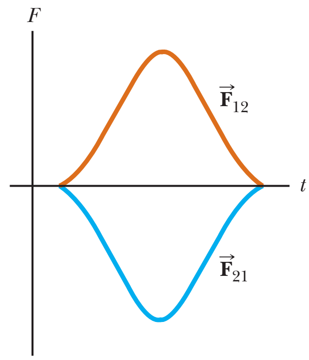


FIGURE 6.8 Force as a function of time for the two colliding particles in Figures 6.6(a) and 6.7. Note that $\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12}$.

Tip 6.1 Momentum Conservation Applies to a *System*!
The momentum of an isolated system is conserved, but not necessarily the momentum of one particle within that system, because other particles in the system may be interacting with it. Apply conservation of momentum to an isolated system *only*.

Conservation of momentum is the principle behind a squid's propulsion system. It propels itself by expelling water at a high velocity.

Mike Severns/Stone/Getty Images

example of recoil, also happens when you fire a gun or shoot an arrow. Conservation of momentum provides a straightforward way to calculate such effects, as the next example shows.

EXAMPLE 6.3 The Archer

Goal Calculate recoil velocity using conservation of momentum.

Problem An archer stands at rest on frictionless ice and fires a 0.500-kg arrow horizontally at 50.0 m/s. (See Fig. 6.9.) The combined mass of the archer and bow is 60.0 kg. With what velocity does the archer move across the ice after firing the arrow?

Strategy Set up the conservation of momentum equation in the horizontal direction and solve for the final velocity of the archer. The system of the archer (including the bow) and the arrow is *not* isolated, because the gravitational and normal forces act on it. These forces, however, are perpendicular to the motion of the system and hence do no work on it.

FIGURE 6.9 (Example 6.3) An archer fires an arrow horizontally to the right. Because he is standing on frictionless ice, he will begin to slide to the left across the ice.

Solution

Write the conservation of momentum equation. Let v_{1f} be the archer's velocity and v_{2f} the arrow's velocity.

$$p_i = p_f$$

$$0 = m_1 v_{1f} + m_2 v_{2f}$$

Substitute $m_1 = 60.0$ kg, $m_2 = 0.500$ kg, and $v_{2f} = 50.0$ m/s, and solve for v_{1f}:

$$v_{1f} = -\frac{m_2}{m_1} v_{2f} = -\left(\frac{0.500\text{ kg}}{60.0\text{ kg}}\right)(50.0\text{ m/s})$$

$$= -0.417\text{ m/s}$$

Remarks The negative sign on v_{1f} indicates that the archer is moving opposite the direction of motion of the arrow, in accordance with Newton's third law. Because the archer is much more massive than the arrow, his acceleration and consequent velocity are much smaller than the acceleration and velocity of the arrow.

Newton's second law, $\Sigma F = ma$, can't be used in this problem because we have no information about the force on the arrow or its acceleration. An energy approach can't be used either, because we don't know how much work is done in pulling the string back or how much potential energy is stored in the bow. Conservation of momentum, however, readily solves the problem.

QUESTION 6.3

Would firing a heavier arrow necessarily increase the recoil velocity? Explain, using the result of Quick Quiz 6.1.

EXERCISE 6.3

A 70.0-kg man and a 55.0-kg woman on ice skates stand facing each other. If the woman pushes the man backwards so that his final speed is 1.50 m/s, at what speed does she recoil?

Answer 1.91 m/s

QUICK QUIZ 6.2 A boy standing at one end of a floating raft that is stationary relative to the shore walks to the opposite end of the raft, away from the shore. As a consequence, the raft (a) remains stationary, (b) moves away from the shore, or (c) moves toward the shore. (*Hint*: Use conservation of momentum.)

6.3 COLLISIONS

We have seen that for any type of collision, the total momentum of the system just before the collision equals the total momentum just after the collision as long as the system may be considered isolated. The total kinetic energy, on the other hand, is generally not conserved in a collision because some of the kinetic energy is converted to internal energy, sound energy, and the work needed to permanently deform the objects involved, such as cars in a car crash. **We define an inelastic collision as a collision in which momentum is conserved, but kinetic energy is not.** The collision of a rubber ball with a hard surface is inelastic, because some of the kinetic energy is lost when the ball is deformed during contact with the surface. **When two objects collide and stick together, the collision is called *perfectly inelastic*.** For example, if two pieces of putty collide, they stick together and move with some common velocity after the collision. If a meteorite collides head on with the Earth, it becomes buried in the Earth and the collision is considered perfectly inelastic. Only in very special circumstances is all the initial kinetic energy lost in a perfectly inelastic collision.

Tip 6.2 Momentum and Kinetic Energy in Collisions

The momentum of an isolated system is conserved in all collisions. However, the kinetic energy of an isolated system is conserved only when the collision is elastic.

An elastic collision is defined as one in which both momentum and kinetic energy are conserved. Billiard ball collisions and the collisions of air molecules with the walls of a container at ordinary temperatures are highly elastic. Macroscopic collisions such as those between billiard balls are only approximately elastic, because some loss of kinetic energy takes place—for example, in the clicking sound when two balls strike each other. Perfectly elastic collisions do occur, however, between atomic and subatomic particles. Elastic and perfectly inelastic collisions are *limiting* cases; most actual collisions fall into a range in between them.

Tip 6.3 Inelastic vs. Perfectly Inelastic Collisions

If the colliding particles stick together, the collision is perfectly inelastic. If they bounce off each other (and kinetic energy is not conserved), the collision is inelastic.

As a practical application, an inelastic collision is used to detect glaucoma, a disease in which the pressure inside the eye builds up and leads to blindness by damaging the cells of the retina. In this application, medical professionals use a device called a *tonometer* to measure the pressure inside the eye. This device releases a puff of air against the outer surface of the eye and measures the speed of the air after reflection from the eye. At normal pressure, the eye is slightly spongy, and the pulse is reflected at low speed. As the pressure inside the eye increases, the outer surface becomes more rigid, and the speed of the reflected pulse increases. In this way, the speed of the reflected puff of air can measure the internal pressure of the eye.

APPLICATION

Glaucoma Testing

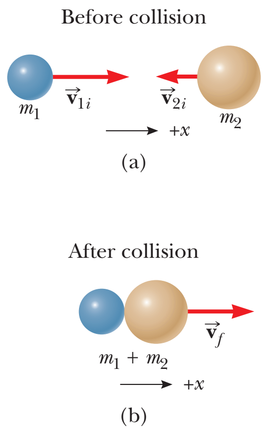
We can summarize the types of collisions as follows:

- In an elastic collision, both momentum and kinetic energy are conserved.
- In an inelastic collision, momentum is conserved but kinetic energy is not.
- In a *perfectly* inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same.

← Elastic collision
← Inelastic collision

In the remainder of this section, we will treat perfectly inelastic collisions and elastic collisions in one dimension.

QUICK QUIZ 6.3 A car and a large truck traveling at the same speed collide head-on and stick together. Which vehicle undergoes the larger change in the magnitude of its momentum? (a) the car (b) the truck (c) the change in the magnitude of momentum is the same for both (d) impossible to determine without more information.



ACTIVE FIGURE 6.10
(a) Before and (b) after a perfectly inelastic head-on collision between two objects.

Perfectly Inelastic Collisions

Consider two objects having masses m_1 and m_2 moving with known initial velocity components v_{1i} and v_{2i} along a straight line, as in Active Figure 6.10. If the two objects collide head-on, stick together, and move with a common velocity component v_f after the collision, then the collision is perfectly inelastic. Because the total momentum of the two-object isolated system before the collision equals the total momentum of the combined-object system after the collision, we can solve for the final velocity using conservation of momentum alone:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)\, v_f \qquad \textbf{[6.8]}$$

Final velocity of two objects in a one-dimensional perfectly inelastic collision →

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} \qquad \textbf{[6.9]}$$

It's important to notice that v_{1i}, v_{2i}, and v_f represent the x-components of the velocity vectors, so care is needed in entering their known values, particularly with regard to signs. For example, in Active Figure 6.10, v_{1i} would have a positive value (m_1 moving to the right), whereas v_{2i} would have a negative value (m_2 moving to the left). Once these values are entered, Equation 6.9 can be used to find the correct final velocity, as shown in Examples 6.4 and 6.5.

EXAMPLE 6.4 An SUV Versus a Compact

Goal Apply conservation of momentum to a one-dimensional inelastic collision.

Problem An SUV with mass 1.80×10^3 kg is traveling eastbound at +15.0 m/s, while a compact car with mass 9.00×10^2 kg is traveling westbound at −15.0 m/s. (See Fig. 6.11.) The cars collide head-on, becoming entangled. **(a)** Find the speed of the entangled cars after the collision. **(b)** Find the change in the velocity of each car. **(c)** Find the change in the kinetic energy of the system consisting of both cars.

Strategy The total momentum of the cars before the collision, p_i, equals the total momentum of the cars after the collision, p_f, if we ignore friction and assume the two cars form an isolated system. (This is called the "impulse approximation.") Solve the momentum conservation equation for the final velocity of the entangled cars. Once the velocities are in hand, the other parts can be solved by substitution.

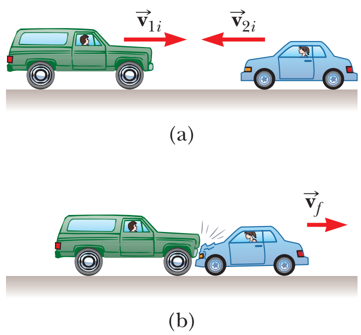


FIGURE 6.11 (Example 6.4)

Solution
(a) Find the final speed after collision.

Let m_1 and v_{1i} represent the mass and initial velocity of the SUV, while m_2 and v_{2i} pertain to the compact. Apply conservation of momentum:

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

Substitute the values and solve for the final velocity, v_f:

$$(1.80 \times 10^3 \text{ kg})(15.0 \text{ m/s}) + (9.00 \times 10^2 \text{ kg})(-15.0 \text{ m/s})$$

$$= (1.80 \times 10^3 \text{ kg} + 9.00 \times 10^2 \text{ kg}) v_f$$

$$v_f = \boxed{+5.00 \text{ m/s}}$$

(b) Find the change in velocity for each car.

Change in velocity of the SUV: $\Delta v_1 = v_f - v_{1i} = 5.00 \text{ m/s} - 15.0 \text{ m/s} = \boxed{-10.0 \text{ m/s}}$

Change in velocity of the compact car: $\Delta v_2 = v_f - v_{2i} = 5.00 \text{ m/s} - (-15.0 \text{ m/s}) = \boxed{20.0 \text{ m/s}}$

(c) Find the change in kinetic energy of the system.

Calculate the initial kinetic energy of the system:

$$KE_i = \tfrac{1}{2}m_1 v_{1i}^2 + \tfrac{1}{2}m_2 v_{2i}^2 = \tfrac{1}{2}(1.80 \times 10^3 \text{ kg})(15.0 \text{ m/s})^2$$
$$+\tfrac{1}{2}(9.00 \times 10^2 \text{ kg})(-15.0 \text{ m/s})^2$$
$$= 3.04 \times 10^5 \text{ J}$$

Calculate the final kinetic energy of the system and the change in kinetic energy, ΔKE.

$$KE_f = \tfrac{1}{2}(m_1 + m_2)v_f^2$$
$$= \tfrac{1}{2}(1.80 \times 10^3 \text{ kg} + 9.00 \times 10^2 \text{ kg})(5.00 \text{ m/s})^2$$
$$= 3.38 \times 10^4 \text{ J}$$
$$\Delta KE = KE_f - KE_i = \boxed{-2.70 \times 10^5 \text{ J}}$$

Remarks During the collision, the system lost almost 90% of its kinetic energy. The change in velocity of the SUV was only 10.0 m/s, compared to twice that for the compact car. This example underscores perhaps the most important safety feature of any car: its mass. Injury is caused by a change in velocity, and the more massive vehicle undergoes a smaller velocity change in a typical accident.

QUESTION 6.4

If the mass of both vehicles were doubled, how would the final velocity be affected? The change in kinetic energy?

EXERCISE 6.4

Suppose the same two vehicles are both traveling eastward, the compact car leading the SUV. The driver of the compact car slams on the brakes suddenly, slowing the vehicle to 6.00 m/s. If the SUV traveling at 18.0 m/s crashes into the compact car, find (a) the speed of the system right after the collision, assuming the two vehicles become entangled, (b) the change in velocity for both vehicles, and (c) the change in kinetic energy of the system, from the instant before impact (when the compact car is traveling at 6.00 m/s) to the instant right after the collision.

Answers (a) 14.0 m/s (b) SUV: $\Delta v_1 = -4.0$ m/s Compact car: $\Delta v_2 = 8.0$ m/s (c) -4.32×10^4 J

EXAMPLE 6.5 The Ballistic Pendulum

Goal Combine the concepts of conservation of energy and conservation of momentum in inelastic collisions.

Problem The ballistic pendulum (Fig. 6.12a, page 172) is a device used to measure the speed of a fast-moving projectile such as a bullet. The bullet is fired into a large block of wood suspended from some light wires. The bullet is stopped by the block, and the entire system swings up to a height h. It is possible to obtain the initial speed of the bullet by measuring h and the two masses. As an example of the technique, assume that the mass of the bullet, m_1, is 5.00 g, the mass of the pendulum, m_2, is 1.000 kg, and h is 5.00 cm. Find the initial speed of the bullet, v_{1i}.

Strategy First, use conservation of momentum and the properties of perfectly inelastic collisions to find the initial speed of the bullet, v_{1i}, in terms of the final velocity of the block–bullet system, v_f. Second, use conservation of energy and the height reached by the pendulum to find v_f. Finally, substitute this value of v_f into the previous result to obtain the initial speed of the bullet.

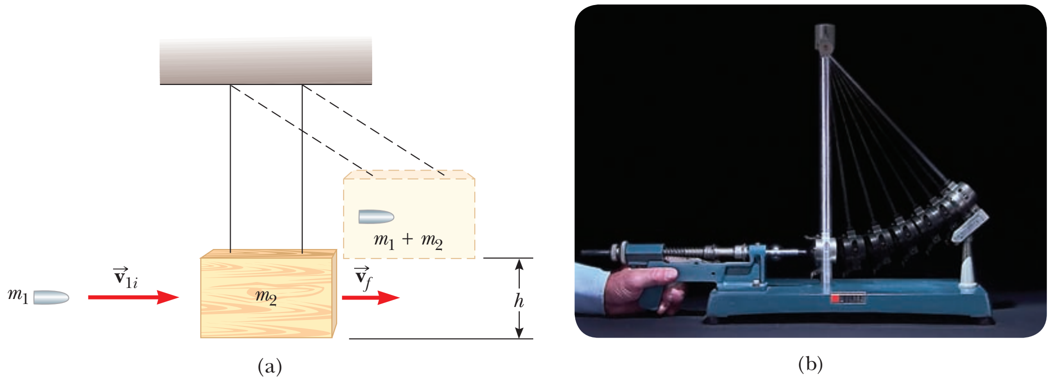


FIGURE 6.12 (Example 6.5) (a) Diagram of a ballistic pendulum. Note that $\vec{\mathbf{v}}_f$ is the velocity of the system just *after* the perfectly inelastic collision. (b) Multiflash photograph of a laboratory ballistic pendulum.

Solution

Use conservation of momentum, and substitute the known masses. Note that $v_{2i} = 0$ and v_f is the velocity of the system (block + bullet) just after the collision.

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_f$$

$$\textbf{(1)} \quad (5.00 \times 10^{-3}\text{ kg})v_{1i} + 0 = (1.005\text{ kg})v_f$$

Apply conservation of energy to the block–bullet system after the collision:

$$(KE + PE)_{\text{after collision}} = (KE + PE)_{\text{top}}$$

Both the potential energy at the bottom and the kinetic energy at the top are zero. Solve for the final velocity of the block–bullet system, v_f:

$$\tfrac{1}{2}(m_1 + m_2)v_f^2 + 0 = 0 + (m_1 + m_2)gh$$

$$v_f^2 = 2gh$$

$$v_f = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(5.00 \times 10^{-2}\text{ m})}$$

$$v_f = 0.990\text{ m/s}$$

Finally, substitute v_f into Equation (1) to find v_{1i}, the initial speed of the bullet:

$$v_{1i} = \frac{(1.005\text{ kg})(0.990\text{ m/s})}{5.00 \times 10^{-3}\text{ kg}} = \boxed{199\text{ m/s}}$$

Remarks Because the impact is inelastic, it would be incorrect to equate the initial kinetic energy of the incoming bullet to the final gravitational potential energy associated with the bullet–block combination. The energy isn't conserved!

QUESTION 6.5

List three ways mechanical energy can be lost from the system in this experiment.

EXERCISE 6.5

A bullet with mass 5.00 g is fired horizontally into a 2.000-kg block attached to a horizontal spring. The spring has a constant 6.00×10^2 N/m and reaches a maximum compression of 6.00 cm. (a) Find the initial speed of the bullet–block system. (b) Find the speed of the bullet.

Answer (a) 1.04 m/s (b) 417 m/s

QUICK QUIZ 6.4 An object of mass m moves to the right with a speed v. It collides head-on with an object of mass $3m$ moving with speed $v/3$ in the opposite direction. If the two objects stick together, what is the speed of the combined object, of mass $4m$, after the collision?
(a) 0 (b) $v/2$ (c) v (d) $2v$

QUICK QUIZ 6.5 A skater is using very low friction rollerblades. A friend throws a Frisbee® at her, on the straight line along which she is coasting.

Describe each of the following events as an elastic, an inelastic, or a perfectly inelastic collision between the skater and the Frisbee. (a) She catches the Frisbee and holds it. (b) She tries to catch the Frisbee, but it bounces off her hands and falls to the ground in front of her. (c) She catches the Frisbee and immediately throws it back with the same speed (relative to the ground) to her friend.

QUICK QUIZ 6.6 In a perfectly inelastic one-dimensional collision between two objects, what condition alone is necessary so that *all* of the original kinetic energy of the system is gone after the collision? (a) The objects must have momenta with the same magnitude but opposite directions. (b) The objects must have the same mass. (c) The objects must have the same velocity. (d) The objects must have the same speed, with velocity vectors in opposite directions.

Elastic Collisions

Now consider two objects that undergo an **elastic head-on collision** (Active Fig. 6.13). In this situation, **both the momentum and the kinetic energy of the system of two objects are conserved.** We can write these conditions as

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad \textbf{[6.10]}$$

and

$$\tfrac{1}{2}m_1 {v_{1i}}^2 + \tfrac{1}{2}m_2 {v_{2i}}^2 = \tfrac{1}{2}m_1 {v_{1f}}^2 + \tfrac{1}{2}m_2 {v_{2f}}^2 \qquad \textbf{[6.11]}$$

where v is positive if an object moves to the right and negative if it moves to the left.

In a typical problem involving elastic collisions, there are two unknown quantities, and Equations 6.10 and 6.11 can be solved simultaneously to find them. These two equations are linear and quadratic, respectively. An alternate approach simplifies the quadratic equation to another linear equation, facilitating solution. Canceling the factor $\frac{1}{2}$ in Equation 6.11, we rewrite the equation as

$$m_1({v_{1i}}^2 - {v_{1f}}^2) = m_2({v_{2f}}^2 - {v_{2i}}^2)$$

Here we have moved the terms containing m_1 to one side of the equation and those containing m_2 to the other. Next, we factor both sides of the equation:

$$m_1(v_{1i} - v_{1f})\,(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})\,(v_{2f} + v_{2i}) \qquad \textbf{[6.12]}$$

Now we separate the terms containing m_1 and m_2 in the equation for the conservation of momentum (Eq. 6.10) to get

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \qquad \textbf{[6.13]}$$

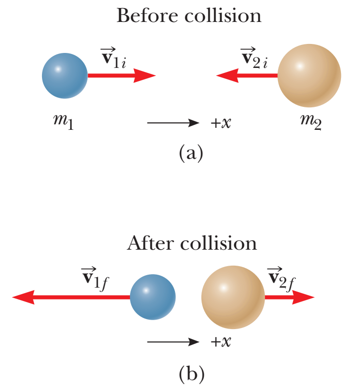
To obtain our final result, we divide Equation 6.12 by Equation 6.13, producing

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

Gathering initial and final values on opposite sides of the equation gives

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f}) \qquad \textbf{[6.14]}$$

This equation, in combination with Equation 6.10, will be used to solve problems dealing with perfectly elastic head-on collisions. According to Equation 6.14, the relative velocity of the two objects before the collision, $v_{1i} - v_{2i}$, equals the negative of the relative velocity of the two objects after the collision, $-(v_{1f} - v_{2f})$. To better understand the equation, imagine that you are riding along on one of the objects. As you measure the velocity of the other object from your vantage point, you will be measuring the relative velocity of the two objects. In your view of the collision, the other object comes toward you and bounces off, leaving the collision with the same speed, but in the opposite direction. This is just what Equation 6.14 states.



ACTIVE FIGURE 6.13
(a) Before and (b) after an elastic head-on collision between two hard spheres.

PROBLEM-SOLVING STRATEGY

ONE-DIMENSIONAL COLLISIONS

The following procedure is recommended for solving one-dimensional problems involving collisions between two objects:

1. **Coordinates.** Choose a coordinate axis that lies along the direction of motion.
2. **Diagram.** Sketch the problem, representing the two objects as blocks and labeling velocity vectors and masses.
3. **Conservation of Momentum.** Write a general expression for the *total* momentum of the system of two objects *before* and *after* the collision, and equate the two, as in Equation 6.10. On the next line, fill in the known values.
4. **Conservation of Energy.** If the collision is elastic, write a general expression for the total energy before and after the collision, and equate the two quantities, as in Equation 6.11 or (preferably) Equation 6.14. Fill in the known values. (***Skip* this step if the collision is *not* perfectly elastic.**)
5. **Solve** the equations simultaneously. Equations 6.10 and 6.14 form a system of two linear equations and two unknowns. If you have forgotten Equation 6.14, use Equation 6.11 instead.

Steps 1 and 2 of the problem-solving strategy are generally carried out in the process of sketching and labeling a diagram of the problem. This is clearly the case in our next example, which makes use of Figure 6.13. Other steps are pointed out as they are applied.

EXAMPLE 6.6 Let's Play Pool

Goal Solve an elastic collision in one dimension.

Problem Two billiard balls of identical mass move toward each other as in Active Figure 6.13. Assume that the collision between them is perfectly elastic. If the initial velocities of the balls are +30.0 cm/s and −20.0 cm/s, what is the velocity of each ball after the collision? Assume friction and rotation are unimportant.

Strategy Solution of this problem is a matter of solving two equations, the conservation of momentum and conservation of energy equations, for two unknowns, the final velocities of the two balls. Instead of using Equation 6.11 for conservation of energy, use Equation 6.14, which is linear, hence easier to handle.

Solution

Write the conservation of momentum equation. Because $m_1 = m_2$, we can cancel the masses, then substitute $v_{1i} = +30.0$ m/s and $v_{2i} = -20.0$ cm/s (Step 3).

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$30.0\text{ cm/s} + (-20.0\text{ cm/s}) = v_{1f} + v_{2f}$$

$$\textbf{(1)}\quad 10.0\text{ cm/s} = v_{1f} + v_{2f}$$

Next, apply conservation of energy in the form of Equation 6.14 (Step 4):

$$\textbf{(2)}\quad v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

$$30.0\text{ cm/s} - (-20.0\text{ cm/s}) = v_{2f} - v_{1f}$$

$$\textbf{(3)}\quad 50.0\text{ cm/s} = v_{2f} - v_{1f}$$

Now solve Equations (1) and (3) simultaneously (Step 5):

$$v_{1f} = \boxed{-20.0\text{ cm/s}} \qquad v_{2f} = \boxed{+30.0\text{ cm/s}}$$

Remarks Notice the balls exchanged velocities—almost as if they'd passed through each other. This is always the case when two objects of equal mass undergo an elastic head-on collision.

QUESTION 6.6

In this example, is it possible to adjust the initial velocities of the balls so that both are at rest after the collision? Explain.

EXERCISE 6.6

Find the final velocity of the two balls if the ball with initial velocity $v_{2i} = -20.0$ cm/s has a mass equal to one-half that of the ball with initial velocity $v_{1i} = +30.0$ cm/s.

Answer $v_{1f} = -3.33$ cm/s; $v_{2f} = +46.7$ cm/s

EXAMPLE 6.7 Two Blocks and a Spring

Goal Solve an elastic collision involving spring potential energy.

Problem A block of mass $m_1 = 1.60$ kg, initially moving to the right with a velocity of $+4.00$ m/s on a frictionless horizontal track, collides with a massless spring attached to a second block of mass $m_2 = 2.10$ kg moving to the left with a velocity of -2.50 m/s, as in Figure 6.14a. The spring has a spring constant of 6.00×10^2 N/m. **(a)** Determine the velocity of block 2 at the instant when block 1 is moving to the right with a velocity of $+3.00$ m/s, as in Figure 6.14b. **(b)** Find the compression of the spring.

Strategy We identify the system as the two blocks and the spring. Write down the conservation of momentum equations, and solve for the final velocity of block 2, v_{2f}. Then use conservation of energy to find the compression of the spring.

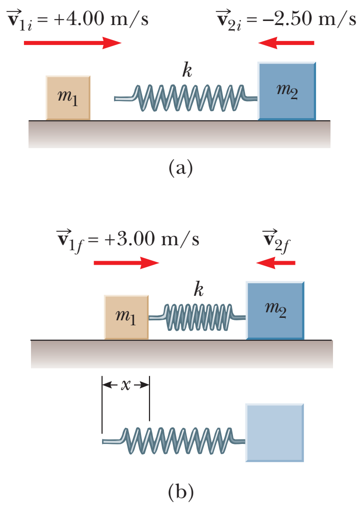


FIGURE 6.14 (Example 6.7)

Solution

(a) Find the velocity v_{2f} when block 1 has velocity $+3.00$ m/s.

Write the conservation of momentum equation for the system and solve for v_{2f}:

$$\textbf{(1)} \quad m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{2f} = \frac{m_1 v_{1i} + m_2 v_{2i} - m_1 v_{1f}}{m_2}$$

$$= \frac{(1.60\text{ kg})(4.00\text{ m/s}) + (2.10\text{ kg})(-2.50\text{ m/s}) - (1.60\text{ kg})(3.00\text{ m/s})}{2.10\text{ kg}}$$

$$v_{2f} = \boxed{-1.74\text{ m/s}}$$

(b) Find the compression of the spring.

Use energy conservation for the system, noticing that potential energy is stored in the spring when it is compressed a distance x:

$$E_i = E_f$$

$$\tfrac{1}{2}m_1 v_{1i}^{\,2} + \tfrac{1}{2}m_2 v_{2i}^{\,2} + 0 = \tfrac{1}{2}m_1 v_{1f}^{\,2} + \tfrac{1}{2}m_2\, v_{2f}^{\,2} + \tfrac{1}{2}kx^2$$

Substitute the given values and the result of part (a) into the preceding expression, solving for x:

$$x = \boxed{0.173\text{ m}}$$

Remarks The initial velocity component of block 2 is -2.50 m/s because the block is moving to the left. The negative value for v_{2f} means that block 2 is still moving to the left at the instant under consideration.

QUESTION 6.7

Is it possible for both blocks to come to rest while the spring is being compressed? Explain. *Hint:* Look at the momentum in Equation (1).

EXERCISE 6.7

Find (a) the velocity of block 1 and (b) the compression of the spring at the instant that block 2 is at rest.

Answer (a) 0.719 m/s to the right (b) 0.251 m

6.4 GLANCING COLLISIONS

In Section 6.2 we showed that the total linear momentum of a system is conserved when the system is isolated (that is, when no external forces act on the system). For a general collision of two objects in three-dimensional space, the conservation of momentum principle implies that the total momentum of the system in each direction is conserved. However, an important subset of collisions takes place in a plane. The game of billiards is a familiar example involving multiple collisions of objects moving on a two-dimensional surface. We restrict our attention to a single two-dimensional collision between two objects that takes place in a plane, and ignore any possible rotation. For such collisions, we obtain two component equations for the conservation of momentum:

$$m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx}$$

$$m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy}$$

We must use three subscripts in this general equation, to represent, respectively, (1) the object in question, and (2) the initial and final values of the components of velocity.

Now, consider a two-dimensional problem in which an object of mass m_1 collides with an object of mass m_2 that is initially at rest, as in Active Figure 6.15. After the collision, object 1 moves at an angle θ with respect to the horizontal, and object 2 moves at an angle ϕ with respect to the horizontal. This is called a *glancing* collision. Applying the law of conservation of momentum in component form, and noting that the initial y-component of momentum is zero, we have

$$x\text{-component:} \quad m_1v_{1i} + 0 = m_1v_{1f}\cos\theta + m_2v_{2f}\cos\phi \qquad \textbf{[6.15]}$$

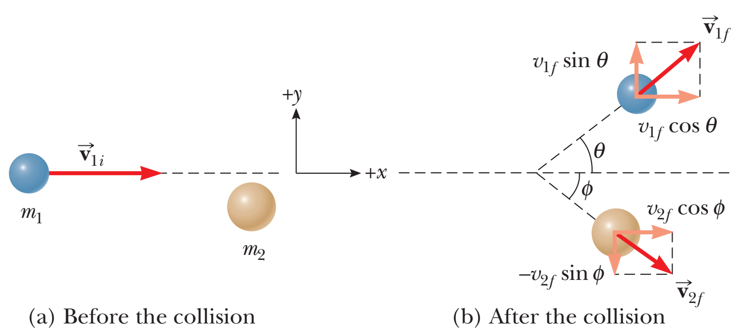
$$y\text{-component:} \quad 0 + 0 = m_1v_{1f}\sin\theta - m_2v_{2f}\sin\phi \qquad \textbf{[6.16]}$$

If the collision is elastic, we can write a third equation, for conservation of energy, in the form

$$\tfrac{1}{2}m_1v_{1i}^{\,2} = \tfrac{1}{2}m_1v_{1f}^{\,2} + \tfrac{1}{2}m_2v_{2f}^{\,2} \qquad \textbf{[6.17]}$$

If we know the initial velocity v_{1i} and the masses, we are left with four unknowns (v_{1f}, v_{2f}, θ, and ϕ). Because we have only three equations, one of the four remaining quantities must be given in order to determine the motion after the collision from conservation principles alone.

If the collision is inelastic, the kinetic energy of the system is *not* conserved, and Equation 6.17 does *not* apply.



ACTIVE FIGURE 6.15
(a) Before and (b) after a glancing collision between two balls.

PROBLEM-SOLVING STRATEGY

TWO-DIMENSIONAL COLLISIONS

To solve two-dimensional collisions, follow this procedure:

1. **Coordinate Axes.** Use both x- and y-coordinates. It's convenient to have either the x-axis or the y-axis coincide with the direction of one of the initial velocities.
2. **Diagram.** Sketch the problem, labeling velocity vectors and masses.
3. **Conservation of Momentum.** Write a separate conservation of momentum equation for each of the x- and y-directions. In each case, the total initial momentum in a given direction equals the total final momentum in that direction.
4. **Conservation of Energy.** If the collision is elastic, write a general expression for the total energy before and after the collision, and equate the two expressions, as in Equation 6.11. Fill in the known values. **(*Skip* this step if the collision is *not* perfectly elastic.)** The energy equation can't be simplified as in the one-dimensional case, so a quadratic expression such as Equation 6.11 or 6.17 must be used when the collision is elastic.
5. **Solve** the equations simultaneously. There are two equations for inelastic collisions and three for elastic collisions.

EXAMPLE 6.8 Collision at an Intersection

Goal Analyze a two-dimensional inelastic collision.

Problem A car with mass 1.50×10^3 kg traveling east at a speed of 25.0 m/s collides at an intersection with a 2.50×10^3-kg van traveling north at a speed of 20.0 m/s, as shown in Figure 6.16. Find the magnitude and direction of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together) and assuming that friction between the vehicles and the road can be neglected.

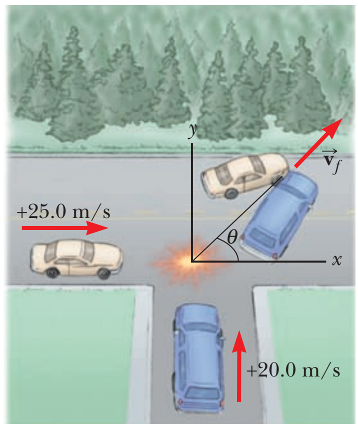


FIGURE 6.16 (Example 6.8) A top view of a perfectly inelastic collision between a car and a van.

Strategy Use conservation of momentum in two dimensions. (Kinetic energy is not conserved.) Choose coordinates as in Figure 6.16. Before the collision, the only object having momentum in the x-direction is the car, while the van carries all the momentum in the y-direction. After the totally inelastic collision, both vehicles move together at some common speed v_f and angle θ. Solve for these two unknowns, using the two components of the conservation of momentum equation.

Solution

Find the x-components of the initial and final total momenta:

$$\sum p_{xi} = m_{\text{car}} v_{\text{car}} = (1.50 \times 10^3 \text{ kg})(25.0 \text{ m/s})$$
$$= 3.75 \times 10^4 \text{ kg}\cdot\text{m/s}$$
$$\sum p_{xf} = (m_{\text{car}} + m_{\text{van}}) v_f \cos\theta = (4.00 \times 10^3 \text{ kg}) v_f \cos\theta$$

Set the initial x-momentum equal to the final x-momentum:

$$\textbf{(1)} \quad 3.75 \times 10^4 \text{ kg}\cdot\text{m/s} = (4.00 \times 10^3 \text{ kg}) v_f \cos\theta$$

Find the y-components of the initial and final total momenta:

$$\sum p_{iy} = m_{\text{van}} v_{\text{van}} = (2.50 \times 10^3 \text{ kg})(20.0 \text{ m/s})$$
$$= 5.00 \times 10^4 \text{ kg}\cdot\text{m/s}$$
$$\sum p_{fy} = (m_{\text{car}} + m_{\text{van}}) v_f \sin\theta = (4.00 \times 10^3 \text{ kg}) v_f \sin\theta$$

Set the initial y-momentum equal to the final y-momentum: **(2)** $5.00 \times 10^4\ \text{kg}\cdot\text{m/s} = (4.00 \times 10^3\ \text{kg})v_f \sin\theta$

Divide Equation (2) by Equation (1) and solve for θ:

$$\tan\theta = \frac{5.00 \times 10^4\ \text{kg}\cdot\text{m/s}}{3.75 \times 10^4\ \text{kg}\cdot\text{m}} = 1.33$$

$$\theta = \boxed{53.1^\circ}$$

Substitute this angle back into Equation (2) to find v_f:

$$v_f = \frac{5.00 \times 10^4\ \text{kg}\cdot\text{m/s}}{(4.00 \times 10^3\ \text{kg})\sin 53.1^\circ} = \boxed{15.6\ \text{m/s}}$$

Remark It's also possible to first find the x- and y-components v_{fx} and v_{fy} of the resultant velocity. The magnitude and direction of the resultant velocity can then be found with the Pythagorean theorem, $v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$, and the inverse tangent function $\theta = \tan^{-1}(v_{fy}/v_{fx})$. Setting up this alternate approach is a simple matter of substituting $v_{fx} = v_f \cos\theta$ and $v_{fy} = v_f \sin\theta$ in Equations (1) and (2).

QUESTION 6.8

If the car and van had identical mass and speed, what would the resultant angle have been?

EXERCISE 6.8

A 3.00-kg object initially moving in the positive x-direction with a velocity of $+5.00$ m/s collides with and sticks to a 2.00-kg object initially moving in the negative y-direction with a velocity of -3.00 m/s. Find the final components of velocity of the composite object.

Answer $v_{fx} = 3.00$ m/s; $v_{fy} = -1.20$ m/s

6.5 ROCKET PROPULSION

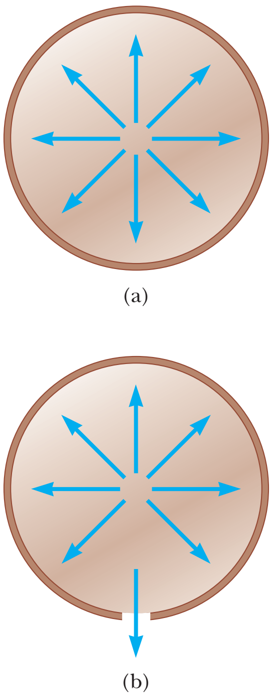


FIGURE 6.17 (a) A rocket reaction chamber without a nozzle has reaction forces pushing equally in all directions, so no motion results. (b) An opening at the bottom of the chamber removes the downward reaction force, resulting in a net upward reaction force.

When ordinary vehicles such as cars and locomotives move, the driving force of the motion is friction. In the case of the car, this driving force is exerted by the road on the car, a reaction to the force exerted by the wheels against the road. Similarly, a locomotive "pushes" against the tracks; hence, the driving force is the reaction force exerted by the tracks on the locomotive. However, a rocket moving in space has no road or tracks to push against. How can it move forward?

In fact, reaction forces also propel a rocket. (You should review Newton's third law, discussed in Chapter 4.) To illustrate this point, we model our rocket with a spherical chamber containing a combustible gas, as in Figure 6.17a. When an explosion occurs in the chamber, the hot gas expands and presses against all sides of the chamber, as indicated by the arrows. Because the sum of the forces exerted on the rocket is zero, it doesn't move. Now suppose a hole is drilled in the bottom of the chamber, as in Figure 6.17b. When the explosion occurs, the gas presses against the chamber in all directions, but can't press against anything at the hole, where it simply escapes into space. Adding the forces on the spherical chamber now results in a net force upwards. Just as in the case of cars and locomotives, this is a reaction force. A car's wheels press against the ground, and the reaction force of the ground on the car pushes it forward. The wall of the rocket's combustion chamber exerts a force on the gas expanding against it. The reaction force of the gas on the wall then pushes the rocket upward.

In a now infamous article in *The New York Times*, rocket pioneer Robert Goddard was ridiculed for thinking that rockets would work in space, where, according to the *Times*, there was nothing to push against. The *Times* retracted, rather belatedly, during the first *Apollo* moon landing mission in 1969. The hot gases are not pushing against anything external, but against the rocket itself—and ironically, rockets actually work *better* in a vacuum. In an atmosphere, the gases have to do work against the outside air pressure to escape the combustion chamber, slowing the exhaust velocity and reducing the reaction force.

At the microscopic level, this process is complicated, but it can be simplified by applying conservation of momentum to the rocket and its ejected fuel. In principle, the solution is similar to that in Example 6.3, with the archer representing the rocket and the arrows the exhaust gases.

Suppose that at some time t, the momentum of the rocket plus the fuel is $(M + \Delta m)v$, where Δm is an amount of fuel about to be burned (Fig. 6.18a). This fuel is traveling at a speed v relative to, say, the Earth, just like the rest of the rocket. During a short time interval Δt, the rocket ejects fuel of mass Δm, and the rocket's speed increases to $v + \Delta v$ (Fig. 6.18b). If the fuel is ejected with exhaust speed v_e *relative to the rocket*, the speed of the fuel relative to the Earth is $v - v_e$. Equating the total initial momentum of the system with the total final momentum, we have

$$(M + \Delta m)v = M(v + \Delta v) + \Delta m(v - v_e)$$

Simplifying this expression gives

$$M\Delta v = v_e \Delta m$$

The increase Δm in the mass of the exhaust corresponds to an equal decrease in the mass of the rocket, so that $\Delta m = -\Delta M$. Using this fact, we have

$$M\Delta v = -v_e \Delta M \qquad \textbf{[6.18]}$$

This result, together with the methods of calculus, can be used to obtain the following equation:

$$v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right) \qquad \textbf{[6.19]}$$

where M_i is the initial mass of the rocket plus fuel and M_f is the final mass of the rocket plus its remaining fuel. This is the basic expression for rocket propulsion; it tells us that the increase in velocity is proportional to the exhaust speed v_e and to the natural logarithm of M_i/M_f. Because the maximum ratio of M_i to M_f for a single-stage rocket is about 10:1, the increase in speed can reach $v_e \ln 10 = 2.3v_e$ or about twice the exhaust speed! For best results, therefore, the exhaust speed should be as high as possible. Currently, typical rocket exhaust speeds are several kilometers per second.

The **thrust** on the rocket is defined as the force exerted on the rocket by the ejected exhaust gases. We can obtain an expression for the instantaneous thrust by dividing Equation 6.18 by Δt:

$$\text{Instantaneous thrust} = Ma = M\frac{\Delta v}{\Delta t} = \left|v_e \frac{\Delta M}{\Delta t}\right| \qquad \textbf{[6.20]}$$

← Rocket thrust

The absolute value signs are used for clarity: In Equation 6.18, $-\Delta M$ is a positive quantity (as is v_e, a speed). Here we see that the thrust increases as the exhaust velocity increases and as the rate of change of mass $\Delta M/\Delta t$ (the burn rate) increases.

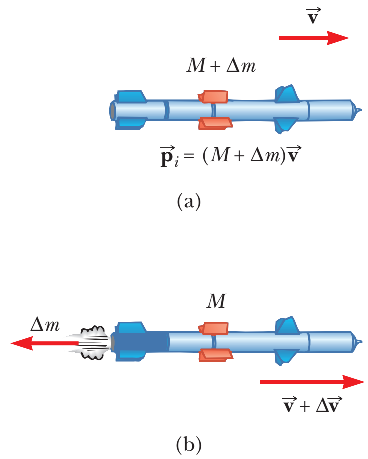


FIGURE 6.18 Rocket propulsion. (a) The initial mass of the rocket and fuel is $M + \Delta m$ at a time t, and the rocket's speed is v. (b) At a time $t + \Delta t$, the rocket's mass has been reduced to M, and an amount of fuel Δm has been ejected. The rocket's speed increases by an amount Δv.

APPLYING PHYSICS 6.2 MULTISTAGE ROCKETS

The current maximum exhaust speed of $v_e =$ 4 500 m/s can be realized with rocket engines fueled with liquid hydrogen and liquid oxygen. But this means that the maximum speed attainable for a given rocket with a mass ratio of 10 is $v_e \ln 10 \approx 10\ 000$ m/s. To reach the Moon, however, requires a change in velocity of over 11 000 m/s. Further, this change must occur while working against gravity and atmospheric friction. How can that be managed without developing better engines?

Explanation The answer is the multistage rocket. By dropping stages, the spacecraft becomes lighter, so that fuel burned later in the mission doesn't have to accelerate mass that no longer serves any purpose. Strap-on boosters, as used by the Space Shuttle and a number of other rockets, such as the *Titan 4* or Russian *Proton*, is a similar concept. The boosters are jettisoned after their fuel is exhausted, so the rocket is no longer burdened by their weight.

EXAMPLE 6.9 Single Stage to Orbit (SSTO)

Goal Apply the velocity and thrust equations of a rocket.

Problem A rocket has a total mass of 1.00×10^5 kg and a burnout mass of 1.00×10^4 kg, including engines, shell, and payload. The rocket blasts off from Earth and exhausts all its fuel in 4.00 min, burning the fuel at a steady rate with an exhaust velocity of $v_e = 4.50 \times 10^3$ m/s. **(a)** If air friction and gravity are neglected, what is the speed of the rocket at burnout? **(b)** What thrust does the engine develop at liftoff? **(c)** What is the initial acceleration of the rocket if gravity is not neglected? **(d)** Estimate the speed at burnout if gravity isn't neglected.

Strategy Although it sounds sophisticated, this problem is mainly a matter of substituting values into the appropriate equations. Part (a) requires substituting values into Equation 6.19 for the velocity. For part (b), divide the change in the rocket's mass by the total time, getting $\Delta M/\Delta t$, then substitute into Equation 6.20 to find the thrust. (c) Using Newton's second law, the force of gravity, and the result of (b), we can find the initial acceleration. For part (d), the acceleration of gravity is approximately constant over the few kilometers involved, so the velocity found in part (b) will be reduced by roughly $\Delta v_g = -gt$. Add this loss to the result of part (a).

Solution

(a) Calculate the velocity at burnout.

Substitute $v_i = 0$, $v_e = 4.50 \times 10^3$ m/s, $M_i = 1.00 \times 10^5$ kg, and $M_f = 1.00 \times 10^4$ kg into Equation 6.19:

$$v_f = v_i + v_e \ln\left(\frac{M_i}{M_f}\right)$$

$$= 0 + (4.5 \times 10^3 \text{ m/s}) \ln\left(\frac{1.00 \times 10^5 \text{ kg}}{1.00 \times 10^4 \text{ kg}}\right)$$

$$v_f = \boxed{1.04 \times 10^4 \text{ m/s}}$$

(b) Find the thrust at liftoff.

Compute the change in the rocket's mass:

$$\Delta M = M_f - M_i = 1.00 \times 10^4 \text{ kg} - 1.00 \times 10^5 \text{ kg}$$
$$= -9.00 \times 10^4 \text{ kg}$$

Calculate the rate at which rocket mass changes by dividing the change in mass by the time (where the time interval equals 4.00 min $= 2.40 \times 10^2$ s):

$$\frac{\Delta M}{\Delta t} = \frac{-9.00 \times 10^4 \text{ kg}}{2.40 \times 10^2 \text{ s}} = -3.75 \times 10^2 \text{ kg/s}$$

Substitute this rate into Equation 6.20, obtaining the thrust:

$$\text{Thrust} = \left| v_e \frac{\Delta M}{\Delta t} \right| = (4.50 \times 10^3 \text{ m/s})(3.75 \times 10^2 \text{ kg/s})$$
$$= \boxed{1.69 \times 10^6 \text{ N}}$$

(c) Find the initial acceleration.

Write Newton's second law, where T stands for thrust, and solve for the acceleration a:

$$Ma = \sum F = T - Mg$$

$$a = \frac{T}{M} - g = \frac{1.69 \times 10^6 \text{ N}}{1.00 \times 10^5 \text{ kg}} - 9.80 \text{ m/s}^2$$
$$= \boxed{7.10 \text{ m/s}^2}$$

(d) Estimate the speed at burnout when gravity is not neglected.

Find the approximate loss of speed due to gravity:

$$\Delta v_g = -g\,\Delta t = -(9.80 \text{ m/s}^2)(2.40 \times 10^2 \text{ s})$$
$$= -2.35 \times 10^3 \text{ m/s}$$

Add this loss to the result of part (b): $v_f = 1.04 \times 10^4 \text{ m/s} - 2.35 \times 10^3 \text{ m/s}$

$$= \boxed{8.05 \times 10^3 \text{ m/s}}$$

Remarks Even taking gravity into account, the speed is sufficient to attain orbit. Some additional boost may be required to overcome air drag.

QUESTION 6.9

What initial normal force would be exerted on an astronaut of mass m in a rocket traveling vertically upward with an acceleration a? Answer symbolically in terms of the positive quantities m, g, and a.

EXERCISE 6.9

A spaceship with a mass of 5.00×10^4 kg is traveling at 6.00×10^3 m/s relative a space station. What mass will the ship have after it fires its engines in order to reach a relative speed of 8.00×10^3 m/s, traveling the same direction? Assume an exhaust velocity of 4.50×10^3 m/s.

Answer 3.21×10^4 kg

SUMMARY

6.1 Momentum and Impulse

The **linear momentum** $\vec{\mathbf{p}}$ of an object of mass m moving with velocity $\vec{\mathbf{v}}$ is defined as

$$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}} \qquad [6.1]$$

Momentum carries units of kg·m/s. The **impulse** $\vec{\mathbf{I}}$ of a constant force $\vec{\mathbf{F}}$ delivered to an object is equal to the product of the force and the time interval during which the force acts:

$$\vec{\mathbf{I}} \equiv \vec{\mathbf{F}}\Delta t \qquad [6.4]$$

These two concepts are unifed in the **impulse–momentum theorem**, which states that the impulse of a constant force delivered to an object is equal to the change in momentum of the object:

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i \qquad [6.5]$$

Solving problems with this theorem often involves estimating speeds or contact times (or both), leading to an average force.

6.2 Conservation of Momentum

When no net external force acts on an isolated system, the total momentum of the system is constant. This principle is called **conservation of momentum**. In particular, if the isolated system consists of two objects undergoing a collision, the total momentum of the system is the same before and after the collision. Conservation of momentum can be written mathematically for this case as

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} \qquad [6.7]$$

Collision and recoil problems typically require finding unknown velocities in one or two dimensions. Each vector component gives an equation, and the resulting equations are solved simultaneously.

6.3 Collisions

In an **inelastic collision**, the momentum of the system is conserved, but kinetic energy is not. In a **perfectly inelastic collision**, the colliding objects stick together. In an **elastic collision**, both the momentum and the kinetic energy of the system are conserved.

A one-dimensional **elastic collision** between two objects can be solved by using the conservation of momentum and conservation of energy equations:

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \qquad [6.10]$$

$$\tfrac{1}{2}m_1{v_{1i}}^2 + \tfrac{1}{2}m_2{v_{2i}}^2 = \tfrac{1}{2}m_1{v_{1f}}^2 + \tfrac{1}{2}m_2{v_{2f}}^2 \qquad [6.11]$$

The following equation, derived from Equations 6.10 and 6.11, is usually more convenient to use than the original conservation of energy equation:

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f}) \qquad [6.14]$$

These equations can be solved simultaneously for the unknown velocities. Energy is not conserved in **inelastic collisions,** so such problems must be solved with Equation 6.10 alone.

6.4 Glancing Collisions

In glancing collisions, conservation of momentum can be applied along two perpendicular directions: an x-axis and a y-axis. Problems can be solved by using the x- and y-components of Equation 6.7. Elastic two-dimensional collisions will usually require Equation 6.11 as well. (Equation 6.14 doesn't apply to two dimensions.) Generally, one of the two objects is taken to be traveling along the x-axis, undergoing a deflection at some angle θ after the collision. The final velocities and angles can be found with elementary trigonometry.

MULTIPLE-CHOICE QUESTIONS

1. A soccer player runs up behind a 0.450-kg soccer ball traveling at 3.20 m/s and kicks it in the same direction as it is moving, increasing its speed to 12.8 m/s. What magnitude impulse did the soccer player deliver to the ball? (a) 2.45 kg·m/s (b) 4.32 kg·m/s (c) 5.61 kg·m/s (d) 7.08 kg·m/s (e) 9.79 kg·m/s

2. A 57.0-g tennis ball is traveling straight at a player at 21.0 m/s. The player volleys the ball straight back at 25.0 m/s. If the ball remains in contact with the racket for 0.060 s, what average force acts on the ball? (a) 22.6 kg·m/s^2 (b) 32.5 kg·m/s^2 (c) 43.7 kg·m/s^2 (d) 72.1 kg·m/s^2 (e) 102 kg·m/s^2

3. A car of mass m traveling at speed v crashes into the rear of a truck of mass $2m$ that is at rest and in neutral at an intersection. If the collision is perfectly inelastic, what is the speed of the combined car and truck after the collision? (a) v (b) $v/2$ (c) $v/3$ (d) $2v$ (e) None of these

4. A small china bowl having kinetic energy E is sliding along a frictionless countertop when a server, with perfect timing, places a rice ball into the bowl as it passes him. If the bowl and rice ball have the same mass, what is the kinetic energy of the system thereafter? (a) $2E$ (b) E (c) $E/2$ (d) $E/4$ (e) $E/8$

5. In a game of billiards, a red billiard ball is traveling in the positive x-direction with speed v and the cue ball is traveling in the negative x-direction with speed $3v$ when the two balls collide head on. Which statement is true concerning their velocities subsequent to the collision? Neglect any effects of spin. (a) red ball: $-v$; cue ball: $3v$ (b) red ball: v; cue ball: $2v$ (c) red ball: $-3v$; cue ball: v (d) red ball: v; cue ball: $3v$ (e) The velocities can't be determined without knowing the mass of each ball.

6. A 5-kg cart moving to the right with a speed of 6 m/s collides with a concrete wall and rebounds with a speed of 2 m/s. Is the change in momentum of the cart (a) 0, (b) 40 kg·m/s, (c) −40 kg·m/s, (d) −30 kg·m/s, or (e) −10 kg·m/s?

7. A 0.10-kg object moving initially with a velocity of 0.20 m/s eastward makes an elastic head-on collision with a 0.15-kg object initially at rest. What is the final velocity of the 0.10-kg object after the collision? (a) 0.16 m/s eastward (b) 0.16 m/s westward (c) 0.040 m/s eastward (d) 0.040 m/s westward (e) None of these

8. A 0.004-kg bullet is fired into a 0.200-kg block of wood at rest on a horizontal surface. After impact, the block with the embedded bullet slides 8.00 m before coming to rest. If the coefficient of friction is 0.400, what is the speed of the bullet before impact? (a) 96 m/s (b) 112 m/s (c) 286 m/s (d) 404 m/s (e) 812 m/s

9. The kinetic energy of a rocket is increased by a factor of eight after its engines are fired, whereas its total mass is reduced by half through the burning of fuel. By what factor is the magnitude of its momentum changed? *Hint:* Use $KE = p^2/2m$. (a) 2 (b) 8 (c) 4 (d) 16 (e) 1

10. If two particles have equal momenta, are their kinetic energies equal? (a) yes, always (b) no, never (c) no, except when their masses are equal (d) no, except when their speeds are the same (e) yes, as long as they move along parallel lines

11. If two particles have equal kinetic energies, are their momenta equal? (a) yes, always (b) no, never (c) yes, as long as their masses are equal (d) yes, if both their masses and directions of motion are the same (e) no, unless they are moving perpendicular to each other

12. A rocket with total mass 3.00×10^5 kg leaves a launch pad at Cape Kennedy, moving vertically with an acceleration of 36.0 m/s^2. If the speed of the exhausted gases is 4.50×10^3 m/s, at what rate is the rocket initially burning fuel? (a) 3.05×10^3 kg/s (b) 2.40×10^3 kg/s (c) 7.50×10^2 kg/s (d) 1.50×10^3 kg/s (e) None of these

CONCEPTUAL QUESTIONS

1. A batter bunts a pitched baseball, blocking the ball without swinging. (a) Can the baseball deliver more kinetic energy to the bat and batter than the ball carries initially? (b) Can the baseball deliver more momentum to the bat and batter than the ball carries initially? Explain each of your answers.

2. Americans will never forget the terrorist attack on September 11, 2001. One commentator remarked that the force of the explosion at the Twin Towers of the World Trade Center was strong enough to blow glass and parts of the steel structure to small fragments. Yet the television coverage showed thousands of sheets of paper floating down, many still intact. Explain how that could be.

3. In perfectly inelastic collisions between two objects, there are events in which all of the original kinetic energy is transformed to forms other than kinetic. Give an example of such an event.

4. If two objects collide and one is initially at rest, is it possible for both to be at rest after the collision? Is it possible for only one to be at rest after the collision? Explain.

5. A ball of clay of mass m is thrown with a speed v against a brick wall. The clay sticks to the wall and stops. Is the principle of conservation of momentum violated in this example?

6. A skater is standing still on a frictionless ice rink. Her friend throws a Frisbee straight at her. In which of the following cases is the largest momentum transferred to the skater? (a) The skater catches the Frisbee and holds onto it. (b) The skater catches the Frisbee momentarily, but then drops it vertically downward. (c) The skater catches the Frisbee, holds it momentarily, and throws it back to her friend.

7. A more ordinary example of conservation of momentum than a rocket ship occurs in a kitchen dishwashing machine. In this device, water at high pressure is forced out of small holes on the spray arms. Use conservation of momentum to explain why the arms rotate, directing water to all the dishes.

8. A large bedsheet is held vertically by two students. A third student, who happens to be the star pitcher on the baseball team, throws a raw egg at the sheet. Explain why the egg doesn't break when it hits the sheet, regardless of its initial speed. (If you try this, make sure the pitcher hits the sheet near its center, and don't allow the egg to fall on the floor after being caught.)

9. Your physical education teacher throws you a tennis ball at a certain velocity, and you catch it. You are now given the following choice: The teacher can throw you a medicine ball (which is much more massive than the tennis ball) with the same velocity, the same momentum, or the same kinetic energy as the tennis ball. Which option would you choose in order to make the easiest catch, and why?

10. If two automobiles collide, they usually do not stick together. Does this mean the collision is elastic? Explain why a head-on collision is likely to be more dangerous than other types of collisions.

11. A sharpshooter fires a rifle while standing with the butt of the gun against his shoulder. If the forward momentum of a bullet is the same as the backward momentum of the gun, why isn't it as dangerous to be hit by the gun as by the bullet?

12. An air bag inflates when a collision occurs, protecting a passenger (the dummy in Figure CQ6.12) from serious injury. Why does the air bag soften the blow? Discuss the physics involved in this dramatic photograph.

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FIGURE CQ6.12

13. In golf, novice players are often advised to be sure to "follow through" with their swing. Why does this make the ball travel a longer distance? If a shot is taken near the green, very little follow-through is required. Why?

14. An open box slides across a frictionless, icy surface of a frozen lake. What happens to the speed of the box as water from a rain shower falls vertically downward into the box? Explain.

PROBLEMS

ENHANCED WebAssign The Problems for this chapter may be assigned online at WebAssign.

1, 2, 3 = straightforward, intermediate, challenging

GP = denotes guided problem

ecp = denotes enhanced content problem

= biomedical application

☐ = denotes full solution available in *Student Solutions Manual/Study Guide*

SECTION 6.1 MOMENTUM AND IMPULSE

1. Calculate the magnitude of the linear momentum for the following cases: (a) a proton with mass 1.67×10^{-27} kg, moving with a speed of 5.00×10^6 m/s; (b) a 15.0-g bullet moving with a speed of 300 m/s; (c) a 75.0-kg sprinter running with a speed of 10.0 m/s; (d) the Earth (mass $= 5.98 \times 10^{24}$ kg) moving with an orbital speed equal to 2.98×10^4 m/s.

2. A stroboscopic photo of a club hitting a golf ball, such as the photo shown in Figure 6.3, was made by Harold Edgerton in 1933. The ball was initially at rest, and the club was shown to be in contact with the club for about 0.002 0 s. Also, the ball was found to end up with a speed of 2.0×10^2 ft/s. Assuming that the golf ball had a mass of 55 g, find the average force exerted by the club on the ball.

☐3. A pitcher claims he can throw a 0.145-kg baseball with as much momentum as a 3.00-g bullet moving with a speed of 1.50×10^3 m/s. (a) What must the baseball's speed be if the pitcher's claim is valid? (b) Which has greater kinetic energy, the ball or the bullet?

4. A 0.10-kg ball is thrown straight up into the air with an initial speed of 15 m/s. Find the momentum of the ball (a) at its maximum height and (b) halfway to its maximum height.

5. A baseball player of mass 84.0 kg running at 6.70 m/s slides into home plate. (a) What magnitude impulse is delivered to the player by friction? (b) If the slide lasts 0.750 s, what average friction force is exerted on the player?

6. ecp Show that the kinetic energy of a particle of mass m is related to the magnitude of the momentum p of that particle by $KE = p^2/2m$. *Note:* This expression is invalid for particles traveling at speeds near that of light.

7. An object has a kinetic energy of 275 J and a momentum of magnitude 25.0 kg·m/s. Find the speed and mass of the object.

8. An estimated force vs. time curve for a baseball struck by a bat is shown in Figure P6.8. From this curve, determine

(a) the impulse delivered to the ball and (b) the average force exerted on the ball.

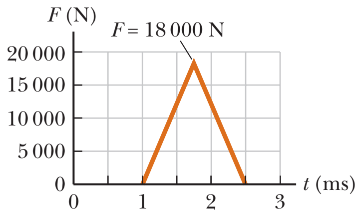

FIGURE P6.8

9. A 0.280-kg volleyball approaches a player horizontally with a speed of 15.0 m/s. The player strikes the ball with her fist and causes the ball to move in the opposite direction with a speed of 22.0 m/s. (a) What impulse is delivered to the ball by the player? (b) If the player's fist is in contact with the ball for 0.060 0 s, find the magnitude of the average force exerted on the player's fist.

10. ecp A man claims he can safely hold on to a 12.0-kg child in a head-on collision with a relative speed of 120-mi/h lasting for 0.10 s as long as he has his seat belt on. (a) Find the magnitude of the average force needed to hold onto the child. (b) Based on the result to part (a), is the man's claim valid? (c) What does the answer to this problem say about laws requiring the use of proper safety devices such as seat belts and special toddler seats?

11. A ball of mass 0.150 kg is dropped from rest from a height of 1.25 m. It rebounds from the floor to reach a height of 0.960 m. What impulse was given to the ball by the floor?

12. A tennis player receives a shot with the ball (0.060 0 kg) traveling horizontally at 50.0 m/s and returns the shot with the ball traveling horizontally at 40.0 m/s in the opposite direction. (a) What is the impulse delivered to the ball by the racket? (b) What work does the racket do on the ball?

13. A car is stopped for a traffic signal. When the light turns green, the car accelerates, increasing its speed from 0 to 5.20 m/s in 0.832 s. What are the magnitudes of the linear impulse and the average total force experienced by a 70.0-kg passenger in the car during the time the car accelerates?

14. A 0.500-kg football is thrown toward the east with a speed of 15.0 m/s. A stationary receiver catches the ball and brings it to rest in 0.020 0 s. (a) What is the impulse delivered to the ball as it's caught? (b) What is the average force exerted on the receiver?

15. The force shown in the force vs. time diagram in Figure P6.15 acts on a 1.5-kg object. Find (a) the impulse of the force, (b) the final velocity of the object if it is initially at rest, and (c) the final velocity of the object if it is initially moving along the x-axis with a velocity of -2.0 m/s.

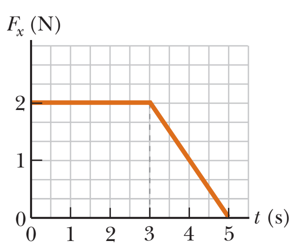

FIGURE P6.15

16. A force of magnitude F_x acting in the x-direction on a 2.00-kg particle varies in time as shown in Figure P6.16. Find (a) the impulse of the force, (b) the final velocity of the particle if it is initially at rest, and (c) the final velocity of the particle if it is initially moving along the x-axis with a velocity of -2.00 m/s.

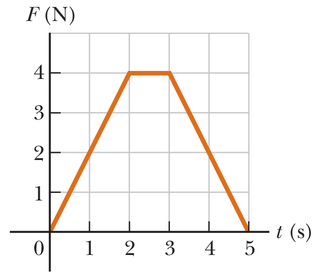

FIGURE P6.16

17. The forces shown in the force vs. time diagram in Figure P6.17 act on a 1.5-kg particle. Find (a) the impulse for the interval from $t = 0$ to $t = 3.0$ s and (b) the impulse for the interval from $t = 0$ to $t = 5.0$ s. (c) If the forces act on a 1.5-kg particle that is initially at rest, find the particle's speed at $t = 3.0$ s and at $t = 5.0$ s.

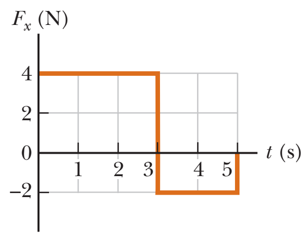

FIGURE P6.17

18. A 3.00-kg steel ball strikes a massive wall at 10.0 m/s at an angle of 60.0° with the plane of the wall. It bounces off the wall with the same speed and angle (Fig. P6.18). If the ball is in contact with the wall for 0.200 s, what is the average force exerted by the wall on the ball?

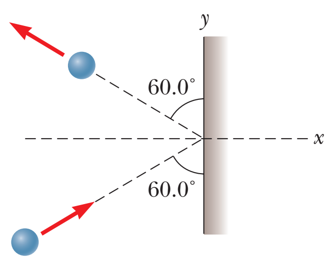

FIGURE P6.18

19. The front 1.20 m of a 1 400-kg car is designed as a "crumple zone" that collapses to absorb the shock of a collision. If a car traveling 25.0 m/s stops uniformly in 1.20 m, (a) how long does the collision last, (b) what is the magnitude of the average force on the car, and (c) what is the acceleration of the car? Express the acceleration as a multiple of the acceleration of gravity.

20. A pitcher throws a 0.15-kg baseball so that it crosses home plate horizontally with a speed of 20 m/s. The ball is hit straight back at the pitcher with a final speed of 22 m/s. (a) What is the impulse delivered to the ball? (b) Find the average force exerted by the bat on the ball if the two are in contact for 2.0×10^{-3} s.

SECTION 6.2 CONSERVATION OF MOMENTUM

21. High-speed stroboscopic photographs show that the head of a 200-g golf club is traveling at 55 m/s just before it strikes a 46-g golf ball at rest on a tee. After the collision, the club head travels (in the same direction) at 40 m/s. Find the speed of the golf ball just after impact.

22. A rifle with a weight of 30 N fires a 5.0-g bullet with a speed of 300 m/s. (a) Find the recoil speed of the rifle. (b) If a 700-N man holds the rifle firmly against his shoulder, find the recoil speed of the man and rifle.

23. A 45.0-kg girl is standing on a 150-kg plank. The plank, originally at rest, is free to slide on a frozen lake, which is a flat, frictionless surface. The girl begins to walk along the plank at a constant velocity of 1.50 m/s to the right relative to the plank. (a) What is her velocity relative to the surface of the ice? (b) What is the velocity of the plank relative to the surface of the ice?

24. A 730-N man stands in the middle of a frozen pond of radius 5.0 m. He is unable to get to the other side because of a lack of friction between his shoes and the ice. To overcome this difficulty, he throws his 1.2-kg physics textbook horizontally toward the north shore at a speed of 5.0 m/s. How long does it take him to reach the south shore?

25. ecp An astronaut in her space suit has a total mass of 87.0 kg, including suit and oxygen tank. Her tether line loses its attachment to her spacecraft while she's on a spacewalk. Initially at rest with respect to her spacecraft, she throws her 12.0-kg oxygen tank away from her spacecraft with a speed of 8.00 m/s to propel herself back toward it (Fig. P6.25). (a) Determine the maximum distance she can be from the craft and still return within 2.00 min (the amount of time the air in her helmet remains breathable). (b) Explain in terms of Newton's laws of motion why this strategy works.

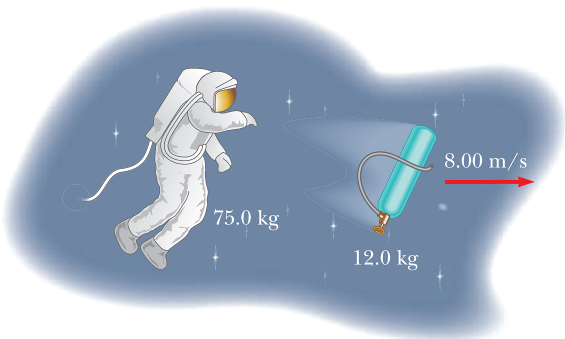

FIGURE P6.25

26. ecp A cannon is mounted on a railroad flatcar, the muzzle elevated to 30.0° and pointed in the direction of the track. The cannon fires a 1.00–metric-ton projectile at 1.00 km/s. (a) If the flatcar and cannon together have a mass of 36.0 metric tons (not including the projectile), what is the initial recoil speed of the flatcar? (b) In this problem, it appears that momentum in the y-direction is not conserved. Explain what happens to it.

27. A 65.0-kg person throws a 0.045 0-kg snowball forward with a ground speed of 30.0 m/s. A second person, with a mass of 60.0 kg, catches the snowball. Both people are on skates. The first person is initially moving forward with a speed of 2.50 m/s, and the second person is initially at rest. What are the velocities of the two people after the snowball is exchanged? Disregard friction between the skates and the ice.

28. ecp Two ice skaters are holding hands at the center of a frozen pond when an argument ensues. Skater A shoves skater B along a horizontal direction. Identify (a) the horizontal forces acting on A and (b) those acting on B. (c) Which force is greater, the force on A or the force on B? (d) Can conservation of momentum be used for the system of A and B? Defend your answer. (e) If A has a mass of 0.900 times that of B, and B begins to move away with a speed of 2.00 m/s, find the speed of A.

SECTION 6.3 COLLISIONS

SECTION 6.4 GLANCING COLLISIONS

29. GP A man of mass $m_1 = 70.0$ kg is skating at $v_1 = 8.00$ m/s behind his wife of mass $m_2 = 50.0$ kg, who is skating at $v_2 = 4.00$ m/s. Instead of passing her, he inadvertently collides with her. He grabs her around the waist, and they maintain their balance. (a) Sketch the problem with before-and-after diagrams, representing the skaters as blocks. (b) Is the collision best described as elastic, inelastic, or perfectly inelastic? Why? (c) Write the general equation for conservation of momentum in terms of m_1, v_1, m_2, v_2, and final velocity v_f. (d) Solve the momentum equation for v_f. (e) Substitute values, obtaining the numerical value for v_f, their speed after the collision.

30. An archer shoots an arrow toward a 300-g target that is sliding in her direction at a speed of 2.50 m/s on a smooth, slippery surface. The 22.5-g arrow is shot with a speed of 35.0 m/s and passes through the target, which is stopped by the impact. What is the speed of the arrow after passing through the target?

31. Gayle runs at a speed of 4.00 m/s and dives on a sled, initially at rest on the top of a frictionless, snow-covered hill. After she has descended a vertical distance of 5.00 m, her brother, who is initially at rest, hops on her back, and they continue down the hill together. What is their speed at the bottom of the hill if the total vertical drop is 15.0 m? Gayle's mass is 50.0 kg, the sled has a mass of 5.00 kg, and her brother has a mass of 30.0 kg.

32. A 75.0-kg ice skater moving at 10.0 m/s crashes into a stationary skater of equal mass. After the collision, the two skaters move as a unit at 5.00 m/s. Suppose the average force a skater can experience without breaking a bone is 4 500 N. If the impact time is 0.100 s, does a bone break?

33. A railroad car of mass 2.00×10^4 kg moving at 3.00 m/s collides and couples with two coupled railroad cars, each of the same mass as the single car and moving in the same direction at 1.20 m/s. (a) What is the speed of the three coupled cars after the collision? (b) How much kinetic energy is lost in the collision?

34. ecp A railroad car of mass M moving at a speed v_1 collides and couples with two coupled railroad cars, each of the same mass M and moving in the same direction at a speed v_2. (a) What is the speed v_f of the three coupled cars after the collision in terms of v_1 and v_2? (b) How much kinetic energy is lost in the collision? Answer in terms of M, v_1, and v_2.

35. ecp Consider the ballistic pendulum device discussed in Example 6.5 and illustrated in Figure 6.12. (a) Determine the ratio of the momentum immediately after the collision to the momentum immediately before the collision. (b) Show that the ratio of the kinetic energy immediately after the collision to the kinetic energy immediately before the collision is $m_1/(m_1 + m_2)$.

36. A 7.0-g bullet is fired into a 1.5-kg ballistic pendulum. The bullet emerges from the block with a speed of 200 m/s, and the block rises to a maximum height of 12 cm. Find the initial speed of the bullet.

37. In a Broadway performance, an 80.0-kg actor swings from a 3.75-m-long cable that is horizontal when he starts. At the bottom of his arc, he picks up his 55.0-kg costar in an inelastic collision. What maximum height do they reach after their upward swing?

38. Two shuffleboard disks of equal mass, one orange and the other yellow, are involved in a perfectly elastic glancing collision. The yellow disk is initially at rest and is struck by the orange disk moving initially to the right at 5.00 m/s. After the collision, the orange disk moves in a direction that makes an angle of 37.0° with its initial direction. Meanwhile, the velocity vector of the yellow disk is perpendicular to the postcollision velocity vector of the orange disk. Determine the speed of each disk after the collision.

39. A 0.030-kg bullet is fired vertically at 200 m/s into a 0.15-kg baseball that is initially at rest. How high does the combined bullet and baseball rise after the collision, assuming the bullet embeds itself in the ball?

40. An 8.00-g bullet is fired into a 250-g block that is initially at rest at the edge of a table of height 1.00 m (Fig. P6.40). The bullet remains in the block, and after the impact the block lands 2.00 m from the bottom of the table. Determine the initial speed of the bullet.

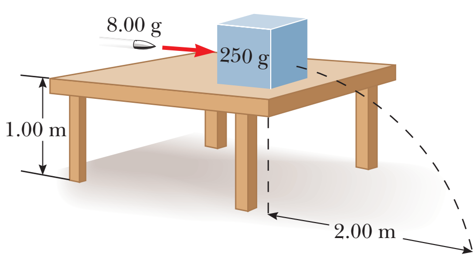

FIGURE P6.40

41. A 12.0-g bullet is fired horizontally into a 100-g wooden block that is initially at rest on a frictionless horizontal surface and connected to a spring having spring constant 150 N/m. The bullet becomes embedded in the block. If the bullet–block system compresses the spring by a maximum of 80.0 cm, what was the speed of the bullet at impact with the block?

42. A 1 200-kg car traveling initially with a speed of 25.0 m/s in an easterly direction crashes into the rear end of a 9 000-kg truck moving in the same direction at 20.0 m/s (Fig. P6.42). The velocity of the car right after the collision is 18.0 m/s to the east. (a) What is the velocity of the truck right after the collision? (b) How much mechanical energy is lost in the collision? Account for this loss in energy.

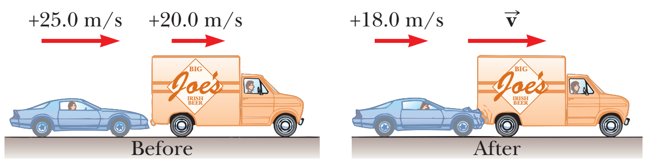

FIGURE P6.42

43. A 5.00-g object moving to the right at 20.0 cm/s makes an elastic head-on collision with a 10.0-g object that is initially at rest. Find (a) the velocity of each object after the collision and (b) the fraction of the initial kinetic energy transferred to the 10.0-g object.

44. GP A space probe, initially at rest, undergoes an internal mechanical malfunction and breaks into three pieces. One piece of mass $m_1 = 48.0$ kg travels in the positive x-direction at 12.0 m/s, and a second piece of mass $m_2 = 62.0$ kg travels in the xy-plane at an angle of 105° at 15.0 m/s. The third piece has mass $m_3 = 112$ kg. (a) Sketch a diagram of the situation, labeling the different masses and their velocities. (b) Write the general expression for conservation of momentum in the x- and y-directions in terms of m_1, m_2, m_3, v_1, v_2, and v_3 and the sines and cosines of the angles, taking θ to be the unknown angle. (c) Calculate the final x-components of the momenta of m_1 and m_2. (d) Calculate the final y-components of the momenta of m_1 and m_2. (e) Substitute the known momentum components into the general equations of momentum for the x- and y-directions, along with the known mass m_3. (f) Solve the two momentum equations for $v_3 \cos\theta$ and $v_3 \sin\theta$, respectively, and use the identity $\cos^2\theta + \sin^2\theta = 1$ to obtain v_3. (g) Divide the equation for $v_3 \sin\theta$ by that for $v_3 \cos\theta$ to obtain $\tan\theta$, then obtain the angle by taking the inverse tangent of both sides. (h) In general, would three such pieces necessarily have to move in the same plane? Why?

45. A 25.0-g object moving to the right at 20.0 cm/s overtakes and collides elastically with a 10.0-g object moving in the same direction at 15.0 cm/s. Find the velocity of each object after the collision.

46. A billiard ball rolling across a table at 1.50 m/s makes a head-on elastic collision with an identical ball. Find the speed of each ball after the collision (a) when the second ball is initially at rest, (b) when the second ball is moving toward the first at a speed of 1.00 m/s, and (c) when the second ball is moving away from the first at a speed of 1.00 m/s.

47. ecp A 90.0-kg fullback running east with a speed of 5.00 m/s is tackled by a 95.0-kg opponent running north with a speed of 3.00 m/s. (a) Why does the tackle constitute a perfectly inelastic collision? (b) Calculate the velocity of the players immediately after the tackle and (c) determine the mechanical energy that is lost as a result of the collision. Where did the lost energy go?

48. Identical twins, each with mass 55.0 kg, are on ice skates and at rest on a frozen lake, which may be taken as frictionless. Twin A is carrying a backpack of mass 12.0 kg. She throws it horizontally at 3.00 m/s to Twin B. Neglecting any gravity effects, what are the subsequent speeds of Twin A and Twin B?

49. A 2 000-kg car moving east at 10.0 m/s collides with a 3 000-kg car moving north. The cars stick together and move as a unit after the collision, at an angle of 40.0° north of east and a speed of 5.22 m/s. Find the speed of the 3 000-kg car before the collision.

50. Two automobiles of equal mass approach an intersection. One vehicle is traveling with velocity 13.0 m/s toward the east, and the other is traveling north with speed v_{2i}. Neither driver sees the other. The vehicles collide in the intersection and stick together, leaving parallel skid marks at an angle of 55.0° north of east. The speed limit for both roads is 35 mi/h, and the driver of the northward-moving vehicle claims he was within the limit when the collision occurred. Is he telling the truth?

51. A billiard ball moving at 5.00 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves at 4.33 m/s at an angle of 30° with respect to the original line of motion. (a) Find the velocity (magnitude and direction) of the second ball after collision. (b) Was the collision inelastic or elastic?

ADDITIONAL PROBLEMS

52. In research in cardiology and exercise physiology, it is often important to know the mass of blood pumped by a person's heart in one stroke. This information can be obtained by means of a *ballistocardiograph*. The instrument works as follows: The subject lies on a horizontal pallet floating on a film of air. Friction on the pallet is negligible. Initially, the momentum of the system is zero. When the heart beats, it expels a mass m of blood into the aorta with speed v, and the body and platform move in the opposite direction with speed V. The speed of the blood can be determined independently (for example, by observing an ultrasound Doppler shift). Assume that the blood's speed is 50.0 cm/s in one typical trial. The mass of the subject plus the pallet is 54.0 kg. The pallet moves 6.00×10^{-5} m in 0.160 s after one heartbeat. Calculate the mass of blood that leaves the heart. Assume that the mass of blood is negligible compared with the total mass of the person. This simplified example illustrates the principle of ballistocardiography, but in practice a more sophisticated model of heart function is used.

53. ecp Most of us know intuitively that in a head-on collision between a large dump truck and a subcompact car, you are better off being in the truck than in the car. Why is this? Many people imagine that the collision force exerted on the car is much greater than that exerted on the truck. To substantiate this view, they point out that the car is crushed, whereas the truck is only dented. This idea of unequal forces, of course, is false; Newton's third law tells us that both objects are acted upon by forces of the same magnitude. The truck suffers less damage because it is made of stronger metal. But what about the two drivers? Do they experience the same forces? To answer this question, suppose that each vehicle is initially moving at 8.00 m/s and that they undergo a perfectly inelastic head-on collision. Each driver has mass 80.0 kg. Including the masses of the drivers, the total masses of the vehicles are 800 kg for the car and 4 000 kg for the truck. If the collision time is 0.120 s, what force does the seat belt exert on each driver?

54. Consider a frictionless track as shown in Figure P6.54. A block of mass $m_1 = 5.00$ kg is released from Ⓐ. It makes a head-on elastic collision at Ⓑ with a block of mass $m_2 = 10.0$ kg that is initially at rest. Calculate the maximum height to which m_1 rises after the collision.

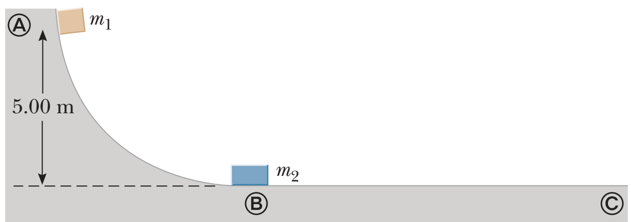

FIGURE P6.54

55. A 2.0-g particle moving at 8.0 m/s makes a perfectly elastic head-on collision with a resting 1.0-g object. (a) Find the speed of each particle after the collision. (b) Find the speed of each particle after the collision if the stationary particle has a mass of 10 g. (c) Find the final kinetic energy of the incident 2.0-g particle in the situations described in parts (a) and (b). In which case does the incident particle lose more kinetic energy?

56. A bullet of mass m and speed v passes completely through a pendulum bob of mass M as shown in Figure P6.56. The bullet emerges with a speed of $v/2$. The pendulum bob is suspended by a stiff rod of length ℓ and negligible mass. What is the minimum value of v such that the bob will barely swing through a complete vertical circle?

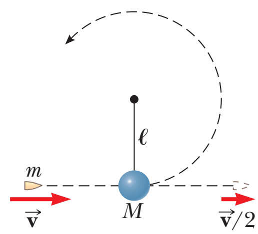

FIGURE P6.56

57. An 80-kg man standing erect steps off a 3.0-m-high diving platform and begins to fall from rest. The man again comes to a rest 2.0 s after reaching the water. What average force did the water exert on him?

58. A 0.400-kg blue bead slides on a curved frictionless wire, starting from rest at point Ⓐ in Figure P6.58 (page 188).

At point Ⓑ, the bead collides elastically with a 0.600-kg blue ball at rest. Find the maximum height the blue ball rises as it moves up the wire.

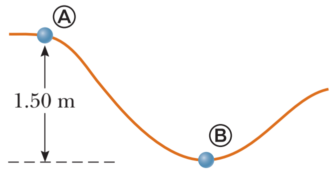

FIGURE P6.58

59. A ball of mass 0.500 kg is dropped from a height of 2.00 m. It bounces against the ground and rises to a height of 1.40 m. If the ball was in contact with the ground for 0.080 0 s, what average force did the ground exert on the ball?

60. An unstable nucleus of mass 1.7×10^{-26} kg, initially at rest at the origin of a coordinate system, disintegrates into three particles. One particle, having a mass of $m_1 = 5.0 \times 10^{-27}$ kg, moves in the positive y-direction with speed $v_1 = 6.0 \times 10^6$ m/s. Another particle, of mass $m_2 = 8.4 \times 10^{-27}$ kg, moves in the positive x-direction with speed $v_2 = 4.0 \times 10^6$ m/s. Find the magnitude and direction of the velocity of the third particle.

61. **ecp** Two blocks of masses m_1 and m_2 approach each other on a horizontal table with the same constant speed, v_0, as measured by a laboratory observer. The blocks undergo a perfectly elastic collision, and it is observed that m_1 stops but m_2 moves opposite its original motion with some constant speed, v. (a) Determine the ratio of the two masses, m_1/m_2. (b) What is the ratio of their speeds, v/v_0?

62. Two blocks of masses $m_1 = 2.00$ kg and $m_2 = 4.00$ kg are each released from rest at a height of 5.00 m on a frictionless track, as shown in Figure P6.62, and undergo an elastic head-on collision. (a) Determine the velocity of each block just before the collision. (b) Determine the velocity of each block immediately after the collision. (c) Determine the maximum heights to which m_1 and m_2 rise after the collision.

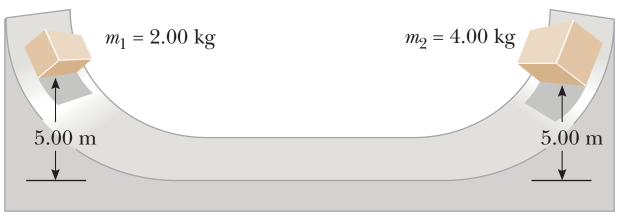

FIGURE P6.62

63. A 0.500-kg block is released from rest at the top of a frictionless track 2.50 m above the top of a table. It then collides elastically with a 1.00-kg object that is initially at rest on the table, as shown in Figure P6.63. (a) Determine the velocities of the two objects just after the collision. (b) How high up the track does the 0.500-kg object travel back after the collision? (c) How far away from the bottom of the table does the 1.00-kg object land, given that the table is 2.00 m high? (d) How far away from the bottom of the table does the 0.500-kg object eventually land?

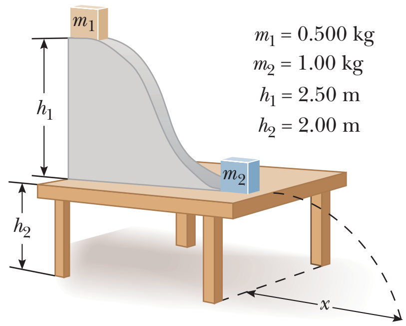

FIGURE P6.63

64. Two objects of masses m and $3m$ are moving toward each other along the x-axis with the same initial speed v_0. The object with mass m is traveling to the left, and the object with mass $3m$ is traveling to the right. They undergo an elastic glancing collision such that m is moving downward after the collision at right angles from its initial direction. (a) Find the final speeds of the two objects. (b) What is the angle θ at which the object with mass $3m$ is scattered?

65. A small block of mass $m_1 = 0.500$ kg is released from rest at the top of a curved wedge of mass $m_2 = 3.00$ kg, which sits on a frictionless horizontal surface as in Figure P6.65a. When the block leaves the wedge, its velocity is measured to be 4.00 m/s to the right, as in Figure P6.65b. (a) What is the velocity of the wedge after the block reaches the horizontal surface? (b) What is the height h of the wedge?

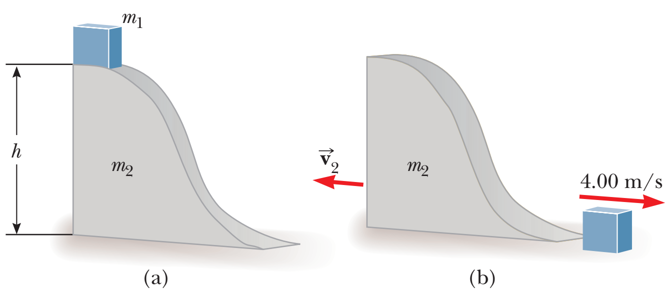

FIGURE P6.65

66. A cue ball traveling at 4.00 m/s makes a glancing, elastic collision with a target ball of equal mass that is initially at rest. The cue ball is deflected so that it makes an angle of 30.0° with its original direction of travel. Find (a) the angle between the velocity vectors of the two balls after the collision and (b) the speed of each ball after the collision.

67. A cannon is rigidly attached to a carriage, which can move along horizontal rails, but is connected to a post by a large spring, initially unstretched and with force constant $k = 2.00 \times 10^4$ N/m, as in Figure P6.67. The cannon

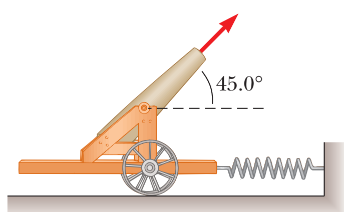

FIGURE P6.67

fires a 200-kg projectile at a velocity of 125 m/s directed 45.0° above the horizontal. (a) If the mass of the cannon and its carriage is 5 000 kg, find the recoil speed of the cannon. (b) Determine the maximum extension of the spring. (c) Find the maximum force the spring exerts on the carriage. (d) Consider the system consisting of the cannon, the carriage, and the shell. Is the momentum of this system conserved during the firing? Why or why not?

68. The "force platform" is a tool that is used to analyze the performance of athletes by measuring the vertical force as a function of time that the athlete exerts on the ground in performing various activities. A simplified force vs. time graph for an athlete performing a standing high jump is shown in Figure P6.68. The athlete started the jump at $t = 0.0$ s. How high did this athlete jump?

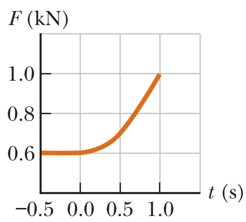

FIGURE P6.68

69. A neutron in a reactor makes an elastic head-on collision with a carbon atom that is initially at rest. (The mass of the carbon nucleus is about 12 times that of the neutron.) (a) What fraction of the neutron's kinetic energy is transferred to the carbon nucleus? (b) If the neutron's initial kinetic energy is 1.6×10^{-13} J, find its final kinetic energy and the kinetic energy of the carbon nucleus after the collision.

70. **ecp** Two blocks collide on a frictionless surface. After the collision, the blocks stick together. Block A has a mass M and is initially moving to the right at speed v. Block B has a mass $2M$ and is initially at rest. System C is composed of both blocks. (a) Draw a free-body diagram for each block at an instant *during* the collision. (b) Rank the magnitudes of the horizontal forces in your diagram. Explain your reasoning. (c) Calculate the change in momentum of block A, block B, and system C. (d) Is kinetic energy conserved in this collision? Explain your answer. (This problem is courtesy of Edward F. Redish. For more such problems, visit http://www.physics.umd.edu/perg.)

71. **ecp** (a) A car traveling due east strikes a car traveling due north at an intersection, and the two move together as a unit. A property owner on the southeast corner of the intersection claims that his fence was torn down in the collision. Should he be awarded damages by the insurance company? Defend your answer. (b) Let the eastward-moving car have a mass of 1 300 kg and a speed of 30.0 km/h and the northward-moving car a mass of 1 100 kg and a speed of 20.0 km/h. Find the velocity after the collision. Are the results consistent with your answer to part (a)?

72. A 60-kg soccer player jumps vertically upwards and heads the 0.45-kg ball as it is descending vertically with a speed of 25 m/s. If the player was moving upward with a speed of 4.0 m/s just before impact, what will be the speed of the ball immediately after the collision if the ball rebounds vertically upwards and the collision is elastic? If the ball is in contact with the player's head for 20 ms, what is the average acceleration of the ball? (Note that the force of gravity may be ignored during the brief collision time.)

73. A tennis ball of mass 57.0 g is held just above a basketball of mass 590 g. With their centers vertically aligned, both balls are released from rest at the same time, to fall through a distance of 1.20 m, as shown in Figure P6.73. (a) Find the magnitude of the downward velocity with which the basketball reaches the ground. (b) Assume that an elastic collision with the ground instantaneously reverses the velocity of the basketball while the tennis ball is still moving down. Next, the two balls meet in an elastic collision. To what height does the tennis ball rebound?

FIGURE P6.73

74. A 20.0-kg toboggan with 70.0-kg driver is sliding down a frictionless chute directed 30.0° below the horizontal at 8.00 m/s when a 55.0-kg woman drops from a tree limb straight down behind the driver. If she drops through a vertical displacement of 2.00 m, what is the subsequent velocity of the toboggan immediately after impact?

75. **ecp** *Measuring the speed of a bullet.* A bullet of mass m is fired horizontally into a wooden block of mass M lying on a table. The bullet remains in the block after the collision. The coefficient of friction between the block and table is μ, and the block slides a distance d before stopping. Find the initial speed v_0 of the bullet in terms of M, m, μ, g, and d.

76. A flying squid (family Ommastrephidae) is able to "jump" off the surface of the sea by taking water into its body cavity and then ejecting the water vertically downward. A 0.85-kg squid is able to eject 0.30 kg of water with a speed of 20 m/s. (a) What will be the speed of the squid immediately after ejecting the water? (b) How high in the air will the squid rise?

77. A 0.30-kg puck, initially at rest on a frictionless horizontal surface, is struck by a 0.20-kg puck that is initially moving along the x-axis with a velocity of 2.0 m/s. After the collision, the 0.20-kg puck has a speed of 1.0 m/s at an angle of $\theta = 53°$ to the positive x-axis. (a) Determine the velocity of the 0.30-kg puck after the collision. (b) Find the fraction of kinetic energy lost in the collision.

78. A 12.0-g bullet is fired horizontally into a 100-g wooden block initially at rest on a horizontal surface. After impact, the block slides 7.5 m before coming to rest. If the coefficient of kinetic friction between block and surface is 0.650, what was the speed of the bullet immediately before impact?